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Virtual wealth protection through virtual money exchange

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ABSTRACT

This article presents pioneering research on *virtual money exchange* (VMX) to protect the wealth generated in virtual worlds. This goal is realized by designing and implementing a VMX system through which virtual money exchange rates between virtual worlds are generated based on a redistribution strategy. Moreover, this strategy aims to re-value the total intrinsic value and the total exchangeable value of virtual currencies at a series of Pareto exchange points. We have designed, implemented, proved and extended a novel *VMX exchange rate algorithm* (VERA) algorithm, which implements the redistribution strategy. In order to observe the behavior of virtual money exchange rate generations, we built a VMX simulator to simulate various cases of virtual money exchange. Experiments on our VMX simulator show that minimum acceptable virtual exchange rates set by exchange requestors have a great impact on the success or failure of a virtual money exchange. This suggests the need for further research to reconcile the contradiction between presenting fair virtual currency exchange rates and achieving low fluctuations of virtual currency exchange rates. Finally, this research contributes to a better understanding of many aspects relating to virtual money, virtual currency exchange and virtual wealth protection. These are important for the future design and implementation of integrated virtual worlds.

1. Introduction

Virtual communities have been of great interest since the dawn of Internet. Soon after different types of virtual communities emerged, fascinating billions of people around the world of all genders, ages, nationalities, cultural backgrounds and educational levels. These users have been called *netizens* (Hauben and Hauben 1996), referring to those who use the Internet to engage in activities of extended virtual social groups. This attracted the research interest of many computer scientists, sociologists, economists and entrepreneurs, who actively shaped a new research field.

A *virtual community* is a technology-supported cyberspace, centered on communication and interaction among participants, resulting in relationships that are built for certain purposes (Lee et al. 2002). A virtual community is a social aggregation emerging from the Internet when enough people carry on public discussions long enough and with sufficient human feeling to form webs of personal relationships in cyberspace (Rheingold 1993). Here similarly, *cyberspace*, a term originally used in William Gibson's science-fiction novel *Neuromancer*, is the name some people use for the conceptual space where words, human relationships, data, wealth, and power are manifested by people using computer-mediated communication technology (Rheingold 1993). As a subcategory of virtual community, a *virtual world* is a virtually-formed common information space in the form of a virtual universe community or a virtual space, where a group of netizens represented as avatars of real-world humans relate to each other, in ways that are characterized by integration, interaction, immersion and interoperability (Cherbakov et al. 2009, Alther 2009).

Technically, a virtual world can be defined as a computer-simulated representation allowing avatars to interconnect and communicate in relatively life-like environments (Ives and Junglas 2008), such as online games and metaverses (Kumar et al. 2008). When two or more virtual worlds are integrated, they constitute an *integrated virtual world*. When many virtual worlds are significantly integrated worldwide, they become *global virtual world*. Now, with the rapid development of many Web-based sciences such as multimedia study, e-commerce and social networking, virtual communities catalyze the evolution of this virtual world, blurring the boundary between the virtual world and the real world.

This article establishes a virtual money exchange regime to protect virtual wealth between virtual worlds or within an integrated virtual world. The importance of building such a virtual currency exchange regime is that it will be possible to freely transfer virtual wealth in the form of virtual money from one virtual world to another. This transferability is essential for virtual wealth protection. Transferability becomes a concern. This is the situation when netizens detect that a virtual world is declining in virtual business,

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implying the potential of virtual wealth loss. Valuable virtual wealth may then be transferred from the worsening virtual world to another more prosperous and stable virtual world to gain more or at least retain the original value of virtual wealth. This behavior is analogous to real-world behavior in similar settings.

A key concern then is how to generate a virtual currency exchange rate at which one virtual currency can be bought for another. A critical research issue encountered here is how to value and measure the worthiness of one virtual currency against another in a fair and transparent way – in essence, how to establish a fair virtual money exchange regime. The challenge of the underlying issue is that a virtual world is often closed in which the value of virtual wealth as measured by the individual virtual currency is arbitrary when compared to another. Guo and Chow (2008) investigated existing virtual money systems and they provide us with a better understanding of this setting. This research shows that most virtual currencies are not interoperable, as virtual worlds are different from the real world, the latter being open to present an integrated global market. In contrast, a virtual world is limited within a virtual community. In the real world, standard baskets of goods, services or currencies can be used as the measurement bases of value comparisons to determine and adjust the exchange rates among real currencies. In contrast, a virtual world is often relatively isolated and its real value of virtual products compared with other virtual worlds is unknown. Thus, before we can build a virtual currency exchange system, we first need to establish a value comparison mechanism between the virtual currencies of virtual worlds.

In establishing such a value comparison mechanism, it is important to investigate the relationship between the existence of a common value system among all netizens for all virtual monies and the problems of a fair virtual money exchange regime. According to Vilfredo Pareto (1906), when following the preference assumptions of a set of individuals, it is optimal that no redistribution of goods can improve the position of one individual without making at least one other individual worse off. This implies that a fair virtual money exchange regime can be established if it complies with the principle of this Pareto optimization.

To research this question, it is first necessary to specify the precise assumptions of common value systems. An important theorem could thus be formulated, which, loosely speaking, asserts that a common value system will exist if two sets of virtual currency exchanges are unique Pareto minimal sets.

Assuming the existence of a common value system, the authors propose a novel *virtual money exchange*(VMX) *regime*, featuring the characteristic of *self-adjustment* so that virtual currency exchange rates could be computer-adjusted or human-adjusted to a Pareto exchange point as the demand and supply of virtual currencies change under the virtual market forces. An additional goal of this research is to provide a basis for future research by conducting a series of experiments on a VMX simulator that is designed and implemented in this article. These experiments will enable us to improve the existing VMX system to support the generation of fairer virtual currency exchange rates for netizens in various virtual worlds.

The rest of the article is organized as follows. Section 2 introduces the concepts of virtual wealth and virtual money exchange. Section 3 reviews the related works on virtual money research and online virtual money exchange shops. Section 4 discusses virtual money value determination practices and the related theories. Section 5 follows and offers a description of a virtual money exchange market and the existence of a common value system. Section 6 proposes a novel VMX model. It describes the technical approach that is used, as well as a virtual exchange rate algorithm (VERA) that describes how virtual currency exchange rates can be generated following different strategies. Section 7 discusses how our VMX simulator was designed and implemented to enable the generation of virtual currency exchange rates. Section 8 presents experiments to simulate virtual currency exchange rate generation. Section 9 offers a discussion on the VMX system from the perspectives of the theory, including Pareto optimization, the Arrow–Debreu model and a double auction system. We conclude in Section 10 with contributions and future research.

2. Virtual wealth and virtual money exchanges

2.1. Virtual wealth

The Internet wave of the 1990s brought forth many novel ideas and features to websites (Wigand 1997). During this time many new applications were created, including forums, blogs, shared files, songs, music, screen savers, software, smilies, online decorating items, game equipment, virtual land and homes, and e-books, among others. We learned that almost any behavior that can be imagined can be digitized and can become a *virtual item*.

In the early years of virtual communities often many virtual items were free. However, web hosting and maintenance efforts imply costs. Covering these costs and, in an effort to make a living, many website operators who provide virtual items began to advertise online. They hoped visitors using free virtual items would click on online ads to compensate the operator for the costs of providing the free virtual items. Award schemes from traditional bonus-based marketing began to play an important role in forming the revenue model for online advertising. Many virtual communities' websites issue bonus points (e.g., ScoreCardRewards.com) and set up various plans to promote the participation of virtual community activities. Netizens are encouraged to browse more web pages, where clicks on ads, views on contents, downloads of items or plays on games are paid for in traditional or electronic money (e.g., Bux.To, LinkBucks.com) or bonus points (e.g., Scour.com). The business thinking behind this scheme is that Internet surfers should share the benefits derived from advertising revenues earned by website operators. Over time, some types of bonus points acquired the label and name of virtual money, and traditional money could be used to buy it in order to purchase virtual items (e.g., LB of Xunlei.com, KinzCash of WebKinz.com and QB of QQ.com). This transformation led to a new subcategory of virtual communities: virtual worlds as various computerized images of real worlds. The development of virtual worlds is centered on the virtualization of world artifacts such as money, commodity, relations and behaviors. These ongoing activities of virtualization have created another business model involving *virtual trade*, which has resulted in *virtual goods* being shaped from virtual items. This practice enables the use of virtual money as the medium of virtual exchange in virtual worlds. By definition, a *virtual good* or *resource* is any virtual-world object or service that increases utility. Utility can be measured in terms of satisfaction, desirability or the usefulness of avatars, and relate to e-books, music files, game equipment, rights to access to virtual resources or payment services that a virtual world provides. These virtual goods and resources gradually evolve to become *virtual wealth*, which points to the power of acquiring any virtual or real goods and the resources for their exchange and use (Smith 1776, p. 28).

Virtual wealth can be measured by *virtual money*, a special type of virtual good that can be used to store the value of other virtual goods like traditional money (Guo and Chow 2008). For such virtual money, as long as a *virtual money issuer* (often the virtual world owner like QQ.com) declares a particular type of *virtual currency* containing a certain amount of value corresponding to certain quantities of virtual goods, this is then used as the medium of exchange and the stored value for virtual goods in the virtual world. It benefits the virtual world by simplifying the cumbersome process of bartering to exchange virtual goods. This further strengthens the position of virtual currency as a symbol of virtual wealth in the sense of stored and measurable common value for that virtual world. When an amount of virtual currency is used to measure a quantity of some virtual good or resource, we also can establish the corresponding *virtual price*.

2.2. Virtual money exchanges

Virtual money is one of the most exciting by-products produced from the evolution of virtual worlds, and it demonstrates the similarity between the virtual and real worlds. The possession of virtual money represents the social and financial position of *virtual inhabitants*, who are avatars of real-world human netizens' images, and shows their richness, diligence, capabilities and other features. When virtual money becomes a means for measuring and storing virtual wealth, and we recognize that virtual wealth should sustain its value as in the real world, then the protection of virtual money value becomes of utmost importance (Guo et al. 2009).

However, research by the authors on existing virtual worlds has shown that there is insufficient understanding of the interactions between various virtual worlds. The existing technical infrastructure does not support different virtual worlds so they can work together to form an integrated virtual world between virtual worlds. Yamaguchi (2004) reports that a virtual currency is valid only in the corresponding virtual world. This is often true. For example, AceBucks of Facebook.com cannot be exchanged to World of Warcraft Gold on WorldofWarcraft.com and QB in QQ.com cannot be converted to LB in Xunlei.com directly from their in-worlds. This reflects the fact that a virtual currency exchange function does not exist between the existing virtual worlds. This implies that virtual inhabitants may lose all of their virtual wealth when their virtual world declines, collapses or goes bankrupt. This is because they have no means of transferring their virtual world-dependent virtual wealth in the form of virtual money to other virtual worlds. A possible solution is to convert virtual money to real money in the real world. But this is still rare. The only example we know of is Linden dollar (L$) in SecondLife.com. L$1000 can be freely converted to about US$4.10. For other virtual worlds, there are three reasons that prevent conversion from virtual to real-world currency:

- *Legal obstacles*. Real-world governments overseeing virtual worlds fear that virtual money will have an unfavorable impact on real-world monetary systems and the economy. For example, China publishes regulations that virtual money can only be converted back to fiat money at the original rate of exchange. So freely buying and selling virtual money is forbidden (www.law-lib.com/law/law_view.asp?id=191034).
- *Monopolistic thought*. Many virtual world operators favor a closed virtual world. In doing so, they can maximize their profits by selling virtual world cards, such as game or gift cards, with prices that are higher than the real exchangeable values. They create virtual world rules in a rather monopolistic fashion also. This constitutes the manipulation of trade between the participants of the virtual world, twisting normal exchange where monopolistic profits are forbidden.
- *Technology limitations*. Different virtual worlds have their own architectures, programming and system design rules. They provide no programming interfaces to real-world monetary or payment systems. This makes it hard for the virtual world to interact with the real world.

In the short-term, the legal obstacles and monopoly issues will not be easy to change. These issues are also beyond the control of the technical design of virtual worlds. The only solution to protect the accumulated virtual wealth of virtual inhabitants is to find a way that the amount of virtual currency from one virtual world can be effectively converted to the virtual currencies of other virtual worlds. This involves explicitly establishing a virtual money exchange regime for the interaction between the different virtual worlds. In this way, if virtual inhabitants believe that a virtual world is collapsing, their virtual wealth can be moved from one to another through a virtual money exchange system.

2.3. Practicability of establishing a virtual money exchange system

A closed virtual world may be desirable in that its owners wish to increase the switching cost of its virtual inhabitants to move to other virtual worlds. This will be beneficial to the owners, since it will permit them to retain their customers and set their own prices. Thus, a virtual money exchange system for virtual wealth protection will not succeed unless it obtains support from existing virtual world owners. This argument seems correct but neglects the fact that an integrated virtual world operating within a viable virtual money exchange regime will provide a much broader virtual market, which will benefit all virtual world owners. This is analogous to the debate about whether we need international trade between countries. Early economists such as David Ricardo (1912) have already demonstrated the comparative advantage of trade between countries.

This analogy is appropriate and lends itself well to explaining how a future integrated virtual world would work. Applying Ricardo's theory, virtual worlds with different virtual products can trade with each other to gain comparative advantage only if a virtual money exchange regime exists between them that supports virtual trade. Proponents of establishing a virtual money exchange regime envision the following benefits: (1) A virtual world's revenue will increase if virtual trade volume increases, which will lead to a boom in the virtual world, increasing other real revenues such as advertising income. (2) An increase in the supply of a world's virtual currency will have the positive effect of increasing real revenue when virtual money is initially bought with real money from either the virtual or the real worlds. (3) The number of virtual inhabitants, including the consumers in the virtual worlds, will increase if virtual trade volume increases, which will make the virtual world more popular, creating additional potential benefits.

In the long run, an integrated virtual world, enabled by a virtual money exchange regime, will permit most virtual worlds to create more value and develop stronger sustainability. So establishing a virtual money exchange regime is necessary to support virtual world participants.

3. Related research

Virtual money is a relatively new research area and is still not well known or sufficiently understood. Even the term *virtual world* is not consistently defined, and the working mechanisms that are present are also not very well understood either. The past couple years have seen more intensive research work in this area. This is due to the popularity of various types of virtual worlds. A dramatically increasing number of people have been attracted to playing games and participating in social networks. Among them, famous examples of virtual money are the Linden Dollar in SecondLife.com, WoW Gold in WorldofWarcraft.com, AceBucks in Facebook.com, QB in QQ.com, and LB in Xunlei.com. In this section, we first review the related work on existing virtual money research. Then, we investigate several virtual money exchange online shops to examine their effectiveness in protecting virtual wealth.

3.1. Research on virtual money

So far many notions of virtual money still mistakenly refer to electronic money. For example, Solomon (1997) describes various types of electronic money using the term *virtual money* (Budd 2000). Mackenzie (2007) viewed virtual money as supporting the network of social relationships that make the exchange possible. Nevertheless, this is still a form of electronic money that carries real money value. In view of the confusion between electronic money and virtual money and the lack of a precise definition of virtual money, Guo and Chow (2008) provided helpful definitions of virtual money, traditional money and electronic money together with a clear distinction among them. They defined virtual money as a type of money virtually created for or from virtual activities by virtual world members in virtual worlds. Its characteristics included: (1) use in the virtual world, (2) non-fiat and fictitious, and (3) commodity, credit or rule-based. Peng and Sun (2009) later offer a similar definition.

Besides the lack of clarity of a virtual money definition, most of the existing research on virtual money leans toward a social, behavioral or economic orientation. With regard to the social aspects, Cikic et al. (2008) argued for the importance of the real value of virtual property and the deficiency of existing virtual environments that lead to fraud and cheating, and illegal use of virtual property. They proposed a set of requirements for building virtual environments. With regard to the behavioral aspects, Guo and Barnes (2007) investigated the determinants of members' behavioral intentions with respect to virtual item transactions in virtual worlds. Their suggested model tried to understand the latent psychological processes that induce transaction-making behavior. Wang and Mainwaring (2008) found that players may abandon, embrace or extend virtual worlds based on the ways game resources can be bought and sold. Users of real money for virtual currencies have raised numerous issues, including realness, trust and fairness. With regard to the economic aspects, Peng and Xu (2009) found that under the current mechanism of Chinese virtual money, speculation is forbidden. The most dominant motivation of users to keep virtual money is for transactional use, not as a store of value. Without the function of storing value, there is no means to determine the value of virtual money though.

The above research has studied the uses and the impacts of virtual money. More technical research on how virtual money can be used and exchanged is still relatively rare though. Guo and Chow (2008) analyzed the types and working mechanisms of existing virtual money systems. They proposed an improved virtual money system model to solve some problems with its real-world use. Other research by Quiané-Ruiz et al. (2008) has suggested that virtual money should be managed to avoid economic problems in peer-to-peer data management systems when queries are made by users and resources to answer them must be allocated.

3.2. Online virtual money exchange shops

The presence of online virtual money exchange shops suggests that there is real-world motivation to study virtual money exchange. An online investigation by the authors found several such shops. For example, GameUSD.com is an information hub that provides price comparisons for various virtual currencies to US dollars and offers exchange rate analyses based on data that are gathered from sampled sellers each day. Also, GameUSD permits users to buy WoW Gold with US dollars. Similarly, IGE.com, one of the world's largest massive multiplayer online role-playing games shops, buys and sells virtual products and virtual currencies in selected massively multiplayer online games in fiat money. Other than trading in fiat money, Sparter.com is a P2P online shop that supports global gamer-to-gamer trading of virtual currencies. Gamers with an existing amount of their game's currency were able to sell it through Sparter.com to other gamers who were looking to buy currency for that specific game, but this service is no longer available. Finally, VirWox.com is an online shop that exchanges Linden dollars with US dollars. It acts as a secondary market for SecondLife.

There are a number of commonalities among the virtual money exchange shops. There is no direct and automated virtual money exchange among virtual currencies. Virtual currencies provided by a single virtual world by different servers are not transferable. Also, no universal exchange mechanism has been established to permit virtual money exchange. These observations signify the high importance that we place on developing a virtual-to-virtual money exchange system.

4. Virtual money value determination

Our review so far has demonstrated that virtual money exchange is important. Nevertheless, how to determine the value of different virtual currencies to form a virtual currency exchange rate between any two virtual currencies is a critical but complicated problem.

4.1. Virtual money bases

To understand what determines the value of virtual money, it is necessary to know the bases for virtual money bases that currently are used in existing virtual worlds. *Money base* is a term related to money supply, which is the amount of money issued in an economy based. In the real world, money is often based on precious metals such as gold and silver in order to have a common measure. This is *commodity-based money*. It also can be based on laws and regulations of a government following some wealth indices. This is *regulation-based legal tender as fiat money*. It also may be based on the credit of an entity that leverages people's trust, which is *credit-based money*. The value of money has relationships with the money base. For example, given the total value V of an economy, if the commodity-based money supply is C, then the value of each unit of commodity money is V/C. Similarly, given a fiat money supply F and credit money T, the unit values of fiat money and credit money will be V/F and V/T.

Virtual money resembles real money in terms of its money bases (Guo and Chow 2008). However, we have observed four layers of worlds, as shown in Fig. 1. A world is an *intrinsic world* if it is perceived by the world itself and is an *extrinsic world* if it is perceived by worlds other than itself. In this layered model, the outer layer worlds provide a given amount of money to the inner layer world as a way of establishing a money supply base. Also, there is a money base relationship between the worlds of the outer layer and the inner layer as follows: *traditional money → electronic money → virtual out-world money → virtual in-world money.*

In the above money-based relationships, *traditional money* refers to money in physical forms, such as notes, coins, checks and bank accounts. *Electronic money* refers to money in electronic

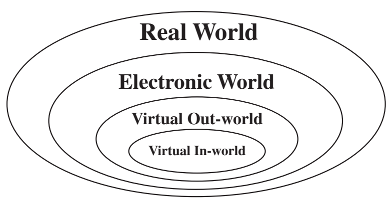


Fig. 1. Layers of worlds.

forms, such as the accounts of e-payment systems like PayPal, or various types of cards including prepaid cards, credit cards and debit cards. This type of money is, in fact, an electronic form of traditional money though. *Virtual out-world money* (or *virtual money form one*) is virtual money that is tightly related to the real and electronic worlds. It is often supplied based on traditional or electronic money. It is used to purchase various types of value-added services that are available in the virtual worlds or by paying subscription fees. For example, consider the virtual world participation time in WorldofWarcraft.com, QB in QQ.com or LB in Xunlei.com. *Virtual in-world money* (or *virtual money form two*) refers to money that is tightly related to virtual in-world activities. It is a type of pure virtual money often used for games (e.g., QGame in QQ.com) and virtual products and services (e.g., Yuanbao of Xunlei.com and Linden dollars for buying and selling virtual products).

By focusing on virtual out-world and virtual in-world monies, as shown in Fig. 2, we have observed four types of virtual money bases: extrinsic world money-based, rule-based, labor-based and credit-based.

Extrinsic world-based virtual money means that the virtual money supply is determined by the given amount of outer-layered worlds. Often, a significant portion of form one virtual money supply is determined by the outer-layer's traditional money and electronic money. For example, the money supply of QB in QQ.com is determined by the quantity of QB sold in Chinese RMB. A good portion of form two virtual money is also outer-layered world money-based. QGame (QQ.com game money) and Yuanbao are supplied based on form one virtual money QB and LB, respectively. *Rule-based virtual money* refers to the virtual money supply that is determined by a virtual world-maker who sets rules to determine the money supply. This includes using rules to turn effort, luck and personal reputation into some amount of virtual money. For example, rules can include the following: when a gamer kills a big "boss," the system gives 100 units of virtual money, or when a gamer meets a specified object such as an "angel," the gamer will obtain 10 units of virtual money. Another possibility is when a gamer is deemed to be honest or reputable by other gamers and the system, which will permit the gamer to gain some amount of virtual money. A significant portion of form two virtual money supply is rule-based, wherein money value is determined by rules of the in-world creators of WoW Gold in WorldofWarcraft.com or AceBucks in Facebook.com.

Labor-based virtual money means that virtual money supply is determined by the amount of time spent in the virtual world. An example is the monthly fee for participation time equivalent to some amount of virtual money. Typical examples are SecondLife monthly fees that can be paid in Linden dollars or QQ.com monthly fees paid in QB. The other example is money awarded to a virtual inhabitant based on the amount of time that the virtual inhabitant stays in a virtual world.

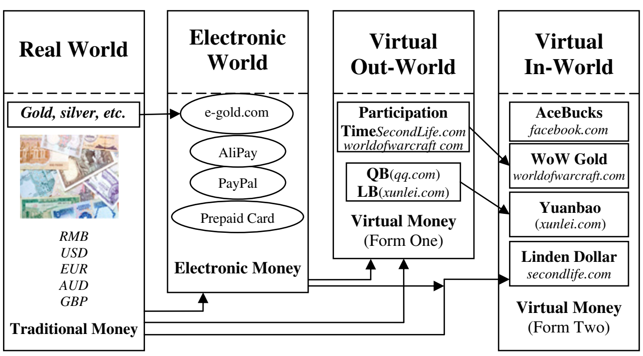


Fig. 2. Virtual money supply bases.

Credit-based virtual money refers to the idea that the virtual money supply is determined by a set of credit indicators, and that the money supply must satisfy those indicators. For example, in SecondLife, there is the practice of managerially pegging Linden dollars to US dollars. In addition, the Linden dollar supply is based on a relatively stable exchange rate against the US dollar. This implies that the Linden dollar supply is based on the credit evaluation of a Linden dollar by comparing it with a US dollar indicator. The adoption of an extrinsic world currency as a credit indicator to adjust virtual money supply is not without problems. When the extrinsic world money reserve approaches zero, the virtual world will have no extrinsic world money to exchange back from the intrinsic world money. When this occurs, the intrinsic world either will reject the virtual money conversion or it will become bankrupt. Based on our observations, most virtual worlds have adopted two or more virtual money bases. For example, QQ.com and Xunlei.com have adopted the money bases of extrinsic worlds, including rules and labor time.

4.2. Value theories

While virtual money bases are used to determine the money supply, it is still not theoretically clear what is the worth of the total virtual money supply in a virtual world. In this subsection, we discuss several theories of value, and attempt to gain a better understanding of the worth of virtual money that is supplied.

4.2.1. Labor theory of value

According to Marx (1887), products that are created on the basis of labor have intrinsic and absolute values, due to the time involved. If the design of a smilie costs four hours of labor and a cartoon picture costs eight hours of labor, the absolute values of the smilie and carton picture can be scaled as four and eight as virtual money in terms of the labor consumed. Similarly, if killing a big "boss" for a sword takes twenty minutes and killing a small "monster" for a knife takes ten minutes, then a sword can be allocated two units of value and a knife one unit of value.

The labor theory of value provides us with a way to measure the different absolute values for varieties of virtual products and services. For example, "paid by ad click" from Bux.to can be evaluated using an amount of value for the time spent. It also provides a method for bartering virtual goods. For example, we can trade two smilies for a cartoon picture based on their absolute values. However, labor effort has different production capabilities (Marx 1887). Labor time for each individual item needs to be averaged to qualify as an absolute criterion for value measurement. The *average labor time* for a virtual good can be computed based on the formula $(\sum_1^n LaborTime_i)/N$, indicating the labor time for the *i*th person and the number of people *N* who are involved in producing the same type of virtual goods.

By applying average labor time, it is possible to establish values for different virtual products and services through comparison. Bartering between virtual products and services also becomes possible and prices can be introduced to label virtual products and services too. The focus is on the labor-based value of virtual money, where value is a function of labor time.

4.2.2. Exchangeable value

An absolute value for average labor time exists in virtual goods. Smith (1776, 0. 28) notes that "labour ...is the real measure of the exchangeable value of all commodities." Its computation is made difficult in reality due to two factors though. People tend to overrate the value of their virtual goods in exchange because of the nature of humans. Measuring the average labor time that is included in virtual goods and finding a way to split this absolute value into shares for each virtual good exchange will not be easy to accomplish.

The exchange of virtual goods thus will not be made very convenient based on measuring their absolute and intrinsic values. Smith observed that money replaced barter and was used to estimate the exchangeable value of commodities (Smith 1776, pp. 29–30). This is because money such as gold and silver are also the products of labor. The trade between commodities and money establishes a price for the commodity, which fluctuates as the valuation of the products change. Thus, the value of a product is not absolute any more but will vary as it is subsequently realized in exchange.

The virtual world can apply prices to evaluate the relative value between two virtual products. Value will change when price changes. Exchangeable value can partly explain rule-based and credit-based virtual money, where value is an evaluation function that is applied by the virtual inhabitants.

4.2.3. Utility as virtual money value

By admitting the products of labor as part of wealth, Mill defines wealth as "all useful or agreeable things which possess exchangeable value; or, in other words, all useful or agreeable things except those which can be obtained, in the quantity desired, without labour or sacrifice" (Mill 1848a, p. 77). He thought that wealth is an instrument of value realization as a means of the attainment of desired things or utility. He refers to "pleasure" (Mill 1848a, p. 76) and "happiness" (Mill 1833, p. 263). Related to utility is the "difficulty of attainment" (Mill 1848b, p. 12) of goods. These factors constitute the conditions for realizing value or "value in exchange" (Mill 1848b, p. 8). Based on these two factors, Mill was thus to explain the fluctuation of exchangeable value of goods around their absolute values. Wealth, this way, can be viewed as "purchasing power" (Mill 1848b, p. 9), but the related realization of value will fluctuate when utility and difficulty of attainment change. This is because when people's utility for a certain good diminishes, the demand for that good decreases, and so their purchasing power for that good increases. Likewise, when the difficulty of attainment of a certain good increases, the supply of that good will fall. Hence, the purchasing power for that good will decrease also. Applying utility theory, we can see that the total value of virtual money in the form of virtual wealth will vary just as the utility of virtual goods in a virtual world will fluctuate. This fluctuation is determined by the desirability of certain virtual goods and the difficulty of finding and obtaining them.

Utility theory (Bentham 1781; Mill 1848a,b) explains what causes the fluctuation of exchangeable values of various virtual goods, which will affect the total value of a virtual world. It can also partly explain credit-based and rule-based virtual money when virtual money value is determined by the total virtual wealth of a virtual world.

4.3. Formation theory for a virtual currency exchange rate

Relating the existing virtual money bases to the value theories, we find that the total value of virtual money for a virtual world compared with other worlds is determined by two factors: an intrinsic-world factor and an extrinsic-world factor. The *extrinsic-world factor* refers to the forces of extrinsic worlds on intrinsic worlds for benefits. The forces include using intrinsic world services (e.g., extrinsic intentions of buying virtual education services in SecondLife), obtaining intrinsic world virtual products (e.g., extrinsic intentions of buying virtual cars and clothes in the QQ.com market) and participating in different kinds of intrinsic world activities (e.g., extrinsic intentions of playing a game in the World of Warcraft). The *intrinsic-world factor* refers to the forces from an intrinsic world to extrinsic worlds that create benefits. The forces include providing virtual services in extrinsic worlds (e.g., intrinsic valuation by SecondLife residents to provide virtual shopping services in Facebook), providing virtual products in extrinsic worlds (e.g., QQ.com's residents' intrinsic valuations of selling virtual products to Xunlei.com) and participating in the activities of extrinsic worlds (e.g., intrinsic valuation by QQ.com gamers to play in the World of Warcraft). These forces meet each other and lead to the changes in the value of virtual money in intrinsic worlds and extrinsic worlds, together with their money supply quantities.

Changes in the value and supply of money in all worlds are illustrated in Fig. 3, where *buying from* and *selling to* activities cause the increase or decrease of money value and the corresponding virtual money quantity supply in the virtual worlds.

Although the total virtual money supply in a virtual world, based on extrinsic world money, rules, and labor or credit, reflects the aggregate value of money there, the total money value is likely to fluctuate, based on the evaluations of intrinsic worlds and extrinsic worlds. Two fundamental concepts apply. The *intrinsic value* of virtual money is extended from the concept of Marx (1887) but also includes the value determined by intrinsic rules, labor time, and extrinsic world money. It reflects the recognition of the total value within a world, and thus it is intrinsic. In contrast, *exchangeable value* is an extension of the concepts of Smith (1776) and Mill (1848a,b) to reflect the total value of a world in the eye of extrinsic worlds. It is determined based on utility (Mill 1848a,b) and given credit by the extrinsic worlds. Based on these definitions, we can infer that there is a commonly accepted value system that is preferred by both the intrinsic worlds and the extrinsic worlds, wherein intrinsic values and exchangeable values are all realized.

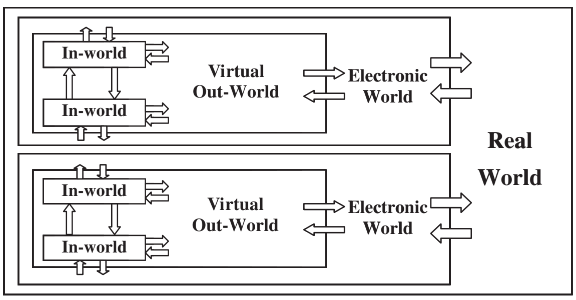
5. A virtual money exchange market

We next describe a virtual money exchange market and prove its existence when a common value system is used in a Pareto optimized system (Pareto 1906).

5.1. Existence of a common value system

A virtual money exchange market will exist if there is a common value system for fair virtual money exchange. To prove this, we first define a virtual money exchange market as follows by applying Pareto points, as described in Geilen et al. (2005):

Definition 1 (*Virtual money exchange market*). A virtual money exchange market is a virtual money exchange system (VMX System) satisfying the following assumptions:



⇨ *Buying from* refers to buying products, services and activities from another world. It also means increasing money supply of selling side to spend on the purchase. The reverse is *selling to,* which increases money supply of buying side.

Fig. 3. Virtual market forces and changes of virtual money supply.

- (Agent). *There is a finite set $\mathcal{A}$ of computer agents on behalf of virtual currency exchange requestors (Player) of intrinsic virtual worlds and extrinsic virtual worlds.*
- (Currency). There is a finite set C of virtual currencies such that each $c_k \in C$ is from a distinct virtual world.
- (Value). For every currency, a value is a set V with a partial order $\leqslant_V$. If the value is not presented, the order is just denoted as $\leqslant$.
- (Preference). Every agent has the preference on the set of value V of smaller values over larger ones. It means that agents always wish to use smaller values to exchange for larger values for any currency exchange for $v_1,\ v_2 \in V,\ v_1 \leqslant v_2$.
- (Exchange space). There is an exchange space S for all currencies such that S is a Cartesian product $V_1 \times V_2 \times \cdots \times V_k \times \cdots \times V_n$ of a finite number of values.
- (Exchange) All agents have an exchange $e^* = (e_1, e_2, \ldots, e_k, \ldots, e_n)$ with each of these an element of exchange space $S = V_1 \times V_2 \times \cdots \times V_k \times \cdots \times V_n$, and e_k denoted by $e^*(V_k)$ or $e^*(k)$. Here, sets $E \subseteq S$ of exchanges represent different value preferences among all agents for realizing different exchange scenarios and how well they meet the intended objectives.

By observation, the above assumptions hold in a virtual economy of an integrated virtual world. Nevertheless, to theoretically prove the existence of the above virtual exchange market, we need first to prove the existence of a *common value system* (*CVS*) in which each virtual currency can be exchanged in a fair value. To prove the existence of a common value system, we offer some definitions as follows:

Definition 2 (*Dominance*). If $e_1^*,\ e_2^* \in S$, then $e_1^* \leqslant e_2^*$ if and only if for every value V_k of $S,\ e_1^*(V_k) \leqslant e_2^*(V_k)$. If $e_1^* \leqslant e_2^*$, the former is said to dominate the latter. The irreflexive variant of $\leqslant$ is <.

Dominance of one exchange over another is a partial order that expresses the fact that the exchange is at least as good, because it is at least as good in each of the individual aspects.

Definition 3 (*Pareto minimal*). A set E of exchanges is said to be Pareto minimal if and only if for any $e_1^*,\ e_2^* \in E,\ e_1^*\ \mathrm{NOT}(<)\ e_2^*$.

Pareto minimality states that a set of exchanges does not contain any strictly dominated exchanges. Exchanges in a set that are not strictly dominated by any other exchange are called *Pareto points*.

Definition 4 (*Set dominance*). A set E_1 of exchanges from exchange space S dominates a set E_2 of exchanges of S, denoted as $E_1 \leqslant E_2$, iff for every $e_2^* \in E_2$, there are some $e_1^* \in E_1$ such that $e_1^* \leqslant e_2^*$.

Definition 5 (*Pareto equivalence*). Two exchange sets E_1 and E_2 from exchange space S are Pareto equivalent, denoted as $E_1 \equiv E_2$, if they dominate each other such that $E_1 \leqslant E_2$ and $E_2 \leqslant E_1$.

Pareto equivalence tells that neither of the two sets contains an exchange that cannot be matched or can be improved upon by the other.

Proposition 1. *If E_1 and E_2 are two Pareto minimal sets of exchanges and $E_1 \equiv E_2$, then $E_1 = E_2$.* The proof can be found in Geilen et al. (2005).

Theorem 1 (*Existence of a unique Pareto minimal set*). *If E is a set of exchanges and $(E, \leqslant)$ is well-ordered, then there is a unique Pareto minimal set U such that $U \equiv E$.* The proof can be found in Geilen et al. (2005).

This theorem shows that every well-ordered set of exchanges has a unique minimal equivalent set. This minimal equivalent set is often called as *Pareto frontier* or *Pareto set*.

Definition 6 (*Virtual currency exchange of intrinsic worlds*). A virtual currency exchange $e_{IV}^* = (iv_1, iv_2, \ldots, iv_k, \ldots, iv_n)$ of intrinsic worlds is an element of the exchange space $V_1 \times V_2 \times \cdots \times V_n$ with $(e_{IV})_k$ also written as iv_k denoted by $e^*_{IV}(V_k)$ and $e^*_{IV}(k)$, where the total intrinsic value is $IV = \sum_1^n iv$ for all n virtual worlds.

In this definition, iv_k is the intrinsic value that the virtual world k thinks that iv_k is the ideal value in exchange for certain values in all of the extrinsic worlds. This is a value assessment from the inside of virtual world k on the value of virtual world k against the values of all non-k virtual worlds.

Definition 7 (*Virtual currency exchange of extrinsic worlds*). A virtual currency exchange $e_{EV}^* = (ev_1, ev_2, \ldots, ev_k, \ldots, ev_n)$ of extrinsic worlds is an element of the exchange space $V_1 \times V_2 \times \cdots \times V_n$ with $(e_{EV})_k$ also written as ev_k denoted by $e_{EV}^*(V_k)$ or $e_{EV}^*(k)$, where the total exchangeable value is $EV = \sum_1^n ev$ for all n virtual worlds.

In this definition, ev_k is the exchangeable value for all virtual worlds except for k. It is a value assessment from outside of virtual world k by all non-k virtual worlds on the value of virtual world k against the values of all non-k virtual worlds.

Theorem 2 (*Existence of a common value system*). *If two sets E_{IV} and E_{EV} of virtual currency exchanges from intrinsic worlds and extrinsic worlds are unique Pareto minimal sets and $E_{IV} \equiv E_{EV}$, then a unique common value system CVS exists between E_{IV} and E_{EV} as a unique Pareto minimal set or a Pareto exchange set or a Pareto exchange point, where the total intrinsic value IV and total exchangeable value EV both are equal to total common value CV, such that it IV = EV = CV.*

The proof is straightforward. Since E_{IV} and E_{EV} are two unique Pareto minimal sets and $E_{IV} \equiv E_{EV}$, then $E_{IV} = E_{EV}$ (Proposition 1). Assuming that $E_{IV} = e_{IV}^* = (iv_1, iv_2, \ldots, iv_n)$ and $E_{EV} = e_{EV}^* = (ev_1, ev_2, \ldots, ev_n)$ following Definitions 6 and 7, then $iv_1 = ev_1,\ iv_2 = ev_2, \ldots, iv_n = ev_n$. Now let the common value system $CVS = e_{CV}^* = (cv_1, cv_2, \ldots, cv_n) = E_{IV} = E_{EV}$. Then we have $iv_1 = ev_1 = cv_1, iv_2 = ev_2 = cv_2, \ldots, iv_n = ev_n = cv_n$. Given $CV = \sum_1^n cv, IV = \sum_1^n iv$ and $EV = \sum_1^n ev$, then $CV = IV = EV$. Thus, there exists a common value system *CVS* referring to both the total intrinsic value and the total exchangeable value.

Theorem 2 essentially claims that a virtual currency exchange system can be best designed and implemented on a Pareto exchange point common value system *CVS* that is fair to both sets of agents in the intrinsic worlds and the extrinsic worlds. This way, any intrinsic value or exchangeable value cannot be improved upon without making some other exchangeable value or intrinsic value worse. A fair common value system implies that the total intrinsic value *IV* must equal the total exchangeable value *EV* at a Pareto exchange point common value system *CVS*. When the common value system *CVS* is implemented in an integrated computer system for mediating different virtual currencies, a virtual money exchange system is created, and will appear as a virtual money exchange market. This shows that a virtual money exchange market should be able to exist.

5.2. Redistribution strategy

Theorem 2 suggests that only one Pareto exchange point common value system exists at some point in time such that the total intrinsic value equals the total exchangeable value. To build a dynamic virtual currency exchange system, we assume that time is continuous. According to Theorem 2, we can always find only one Pareto exchange point common value system *CVS* at any time where all virtual currency exchanges are fair to all agents representing the players in of the worlds. The particular design and implementation of a common value system *CVS* from one time to the next time is called a *redistribution strategy*, which dynamically revalues virtual currency values for all virtual currency exchanges. We define the virtual currency quantity reflecting the intrinsic value as the money supply of self-valued worlds, and define the virtual currency quantity reflecting the exchangeable value as the money demand of non-self-valued worlds. Based on these definitions, the process of continuously computing a set of virtual currency exchange rates along a continuous time line makes it possible to reassess the value for all virtual currencies on a set of Pareto exchange points. The redistribution strategy can be stated more rigorously as follows:

Definition 8 (*Redistribution strategy*). For any dynamic Pareto exchange point, the sum of the money supply of an arbitrary virtual currency in a set of virtual currencies, considered as the intrinsic value of that currency, always is equal to the sum of the money demand on that virtual currency from a set of other currencies, considered as the exchangeable value of that currency. Inequality between the supply and demand in terms of quantity can always be adjusted by a set of floating exchange rates, which guarantees a return to the Pareto exchange point.

After we obtain a set of virtual currency exchange rates, the inequality will arise again until the next Pareto exchange point. The theoretical interpretation of the Pareto exchange point is the total common value presented by both sets of intrinsic worlds and extrinsic worlds. This interpretation is also the theoretical foundation for the formation of virtual currency exchange rates.

6. Virtual money exchange approach

In this section, we describe a *virtual money exchange* (VMX) approach to implement the redistribution strategy. We first present an overview of how virtual currencies are exchanged in a VMX system, then the details of how to generate the exchange rate, as determined by the VERA algorithm. Finally, we discuss an extension of the VERA algorithm.

6.1. VMX system overview

VMX is an approach to virtual currency exchange between virtual worlds. It requires the acquisition of the demand and supply of virtual currencies. This contribution assumes that the needed demand and supply information is available as the inputs of virtual inhabitants. In depth discussion of this information is beyond the scope of this article, and is discussed elsewhere (Guo 2006, 2008).

At a high conceptual level, the VMX approach can be presented in a VMX cross-world process model, as shown in Fig. 4, where a virtual currency exchange request (buying and selling) is issued from A to the VMX system through A's agent. The VMX server then calculates the virtual currency exchange rate, offers it at a Pareto exchange point, and sends this offer back to A through A's agent. If A accepts this offer, A sends its acceptance to the VMX server and VMX will conclude the deal.

In this process model, the cross-world virtual money exchange process is executed by three roles: the player, the agent and the VMX server. A player in a virtual world issues exchange requests and executes exchange orders. An agent is the representative of a player in the VMX system. It is responsible for delivering the messages of exchange between the player and the VMX server. The VMX server is the engine for processing the exchange requests and responses. It also calculates virtual currency exchange rates and offers. Both the VMX server and the agent are part of the VMX system, since it represents a virtual money exchange market, while a player is a virtual inhabitant, similar to many others in virtual worlds.

The process model represents the idea of a direct virtual currency exchange. It implements the redistribution strategy that we discussed earlier with the spirit of "what you received is what others offered." The VMX server and its agent only act as mediators to generate exchange rates based on the real-time collected exchange requests from players in various virtual worlds.

6.2. VMX exchange rate algorithm

In designing the VMX system, the core part is the *VMX exchange rate algorithm* (VERA), which dynamically computes virtual currency exchange rates between virtual currencies. This article only focuses on this core part and will not consider other components, such as agent design issues for linking to a variety of virtual worlds.

6.2.1. The central idea of VERA

VERA serves the virtual currency exchange on behalf of players, who make request, and whose virtual currency has been deposited in an appropriate account. VERA is designed based on the money demand and money supply of all virtual currencies from the players. Applying the redistribution strategy that we discussed earlier, VERA computes the expected virtual currency exchange rates at the Pareto exchange points for money supply and money demand for each virtual currency. To obtain the desired computational results, VERA makes a number of assumptions.

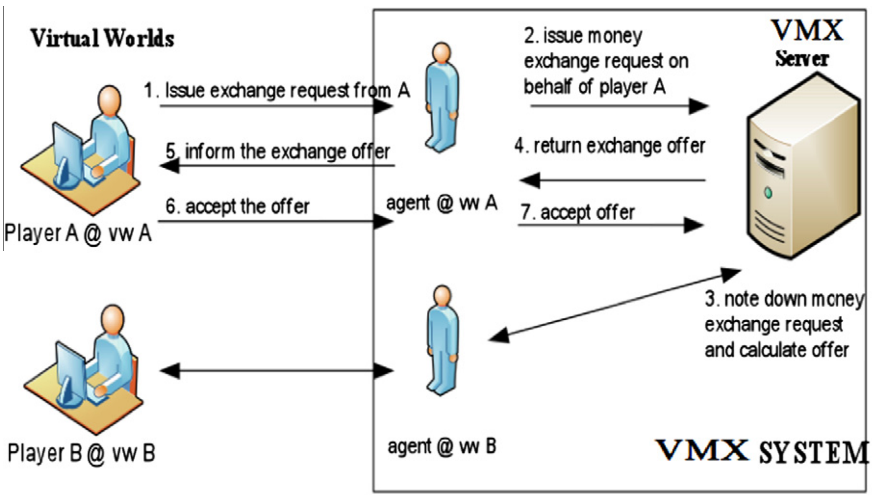


Fig. 4. VMX cross-world process model.

Assumption 1. The total money supply of a virtual currency relative to exchange with other virtual currencies is always equal to the total demand of that virtual currency relative to its demand for exchange of all other virtual currencies. We will prove this assumption in Appendix A.

Assumption 2. Information about money demand and supply for all virtual currencies is always available in practice, where money demand is represented by Buy-Lead and money supply is represented by Sell-Lead. This assumption always holds when the VMX system is accessible to all players.

Based on these two assumptions, for the inverse of the exchange rate between two currencies, and requirement that the supply and demand for the currencies should match at a Pareto exchange point, we have *demand of currency = supply of the currency × exchange rate between the currencies*, with:

- *The Sell-Lead formula* : *demand* $=$ *supply* $\times$ *exchange rate* (1)
- *The Buy-Lead formula* : *supply* $=$ *demand*/*exchange rate* (2)

These formulas are the foundation of the VERA design, from which all transaction *requests* and *offers* are generated.

6.2.2. Assumptions and notation

Let C be the set of virtual currencies that are eligible for exchange. $C = \{c_1, c_2, \ldots, c_n\}$. Based on Eq. (1), the agents of the intrinsic world players (the sellers) provide an arbitrary currency c_x in order to exchange for another currency c_y, which forms the supply of c_x to c_y and is denoted by s_{xy} and denominated in the units of currency c_x. As depicted in Eq. (1), the unknown is the demand, denoted by ud_{xy} and denominated in the units of currency c_y. Based on Eq. (2), agents of the extrinsic world players (the buyers) desire to obtain currency c_x with currency c_y, which is the demand of c_x, denoted by d_{xy} and denominated in the units of currency c_x. As described in Eq. (2), the unknown is the supply, which is denoted by us_{xy} and denominated in the units of currency c_y. Following Assumptions 1 and 2, the total amount of supply of currency c_x is given by $\sum_{y=c_1}^{c_n} s_{xy}$. Likewise, the total amount of demand for currency c_x is given by $\sum_{y=c_1}^{c_n} d_{xy}$. Let e_{xy} be the exchange rate of currency c_x to c_y. Assume there are n currencies, and their exchange rates are denoted by the set E_{xy} such that $E_{xy} = \{e_{11}, e_{12}, \ldots, e_{1n}, \ldots, e_{nn}\}$. The cardinality or number of elements of E_{xy} is $|E_{xy}| = n^2$. Based on this notation, Eqs. (1) and (2) can be rewritten as:

$$ud_{yx} = s_{xy} \cdot e_{xy} \quad \text{(Sell-Lead)} \tag{3}$$

$$us_{yx} = d_{xy} \cdot e_{xy} \quad \text{(Buy-Lead)} \tag{4}$$

Let IV_{c_x} and EV_{c_x} be the intrinsic value and exchangeable value of c_x. As defined before, IV_{c_x} is the total supply of c_x such $IV_{c_x} = \sum_{y=c_1}^{c_n} s_{xy}$ in Sell-Lead of $IV_{c_x} = \sum_{y=c_1}^{c_n} us_{xy}$ in Buy-Lead. EV_{c_x} denotes the total demand of c_x such that $EV_{c_x} = \sum_{y=c_1}^{c_n} ud_{xy}$ in Sell-Lead or $EV_{c_x} = \sum_{y=c_1}^{c_n} d_{xy}$ in Buy-Lead. Based on the redistribution strategy, the demand of any virtual currency should be fulfilled by the supply of that currency such that all currencies on hand are sold at Pareto exchange point at any fair prices, and thus, supply equals demand, stated as follows:

Definition 9. (*Supply s = Demand d*) $\forall c_x \in C$, $s = d$, such that:

$$\sum_{y=c_1}^{c_n} s_{xy} = \sum_{y=c_1}^{c_n} d_{xy} \tag{5}$$

where

$$\sum_{y=c_1}^{c_n} s_{xy} = \sum_{y=c_1}^{c_n} ud_{xy} \quad \text{(Sell-Lead)} \tag{6}$$

$$\sum_{y=c_1}^{c_n} us_{xy} = \sum_{y=c_1}^{c_n} d_{xy} \quad \text{(Buy-Lead)} \tag{7}$$

Note that Eqs. (5)–(7) above denote equality only in terms of the common value of virtual currencies between supply and demand. The common value can be represented in different currency units. Thus, unit conversion is necessary. We will discuss this shortly.

6.2.3. Conditions: when t = 0

Assume that when $t = 0$ there is no request made through the system for the exchange between any virtual currency c_x and c_y such that $c_x \in C$ and $c_y \in C$. Therefore, the amount of supply and demand of each $c_x \in C$ will equal zero. Thus $IV_{c_x} = EV_{c_x} = 0$, which means that each virtual world is isolated and its supplied virtual currency has no exchangeable value. Consequently, no exchange rate exists among different virtual currencies, and we have a null value. The equations below express this idea, and represent the conditions for the Sell-Lead and the Buy-Lead. Thus, we have a pre-condition when $t = 0$ that $\forall x \in C \wedge \forall y \in C$, which leads to:

$$ud_{xy} = 0 \quad \text{and} \quad s_{xy} = 0 \tag{8}$$

$$d_{xy} = 0 \quad \text{and} \quad us_{xy} = 0 \tag{9}$$

There is a also a post-condition when$t = 0$, which leads to:

$$e_{xy} = \text{NULL} \tag{10}$$

In the VERA algorithm's design, the state $t = 0$ is the *initial state* of the VMX system. When the above condition is satisfied, the VMX system will be *initialized*.

6.2.4. Conditions when t = 1, 2, . . ., k, . . ., m

Assume when $t = 1$ and $t_1 - t_0 = f$ milliseconds, there are requests made through VMX system regarding the exchange of any currency c_x and c_y with $c_x \in C$ and $c_y \in C$. Therefore, there exist some s_{xy} and d_{xy} such that both of them are greater than zero. The requests are made through either Buy-Lead or Sell-Lead. All requests issued between $t = k - 1$ and $t = k$ are grouped based on their types of business and currency. For any point in time $t = k(k \in \mathbb{N})$, we can define a cutoff point of the system such that all exchange requests made between $t = k - 1$ and $t = k$ are cleared after the exchange rate computation occurs.

Based on the redistribution strategy in Definition 8 and Theorem 2, $IV_{c_x} = EV_{c_x}$ for all $c_x \in C$, such that each virtual currency has an exchangeable value equaled to its intrinsic value in Eqs. (6) and (7). This leads to having exchange rates among different virtual currencies $e_{xy} \geqslant 0$. Formally, the pre-condition of the VERA algorithm is the same as in Eqs. (8) and (9), while the post-condition is given by:

$$e_{xy} \geqslant 0 \tag{11}$$

To satisfy the above pre-condition, s_{xy} and d_{xy} can be retrieved from the requests made by various players who issue exchange requests in form of Buy-Lead or Sell-Lead.

6.2.5. Exchange rate computation

Next we compute the exchange rate. As we stated before, the cardinality of $E_{xy} = n^2$. Here n denotes the number of virtual currencies that participate in the exchange. Based on Eqs. (1), (2) and (5), we can compute e_{xy}. Nevertheless, we also face two problems: computation of the sum of demand $\sum_{y=c_1}^{c_n} d_{xy}$ and supply $\sum_{y=c_1}^{c_n} us_{xy}$ involves different virtual currencies, where amounts cannot simply be summed due to the different units; and at least n^2 equations must be available and solved to compute all the e_{xy}.

To solve the two problems, an intermediate currency called CONEY is introduced in computation. Theoretically, CONEY holds the total common value of total intrinsic value and total exchangeable value such that the total intrinsic value IV = the total exchangeable value EV = the total value of CONEY at the Pareto exchange point (Theorem 2). The main idea for how we solve this problem is that we convert all the different currencies into a single currency CONEY and also reduce the number of equations that are required. CONEY is meaningful only during the virtual currency exchange rate computation at the Pareto exchange point.

Let be the exchange rate of currency c_x to CONEY c_o and e_{yo} be the exchange rate of currency to CONEY c_o. Then, the exchange rate of currency x to currency y is:

$$e_{xy} = e_{xo}/e_{yo} \tag{12}$$

Therefore, the variables (exchange rates) become $E'_{xo} = \{e_{1o}, e_{2o}, \ldots, e_{no}\}$, where its cardinality is $|E'_{xo}|$. To find out all exchange rates, we have to build n equations, which will be deduced later.

Let z_p be the amount of CONEY supply converted from the supply of virtual currency c_x, which is normally found in Sell-Lead business and let z_q be amount of CONEY demand converted from the demand on virtual currency c_x. Then, we have:

$$z_p = s_{xy} \cdot e_{xo} \quad (\text{Sell} - \text{Lead}) \tag{13}$$

or

$$z_q = d_{xy} \cdot e_{xo} \quad (\text{Buy} - \text{Lead}) \tag{14}$$

Let $z_{p'}$ be the amount of CONEY demand converted from the unknown demand of virtual currency c_x found in Sell-Lead and let $z_{q'}$ be the amount of CONEY supply converted from the unknown supply of virtual currency c_x as found in Buy-Lead, we have:

$$z_{p'} = ud_{xy} \cdot e_{xo} \quad (\text{Sell} - \text{Lead}) \tag{15}$$

or

$$z_{q'} = us_{xy} \cdot e_{xo} \quad (\text{Buy} - \text{Lead}) \tag{16}$$

As we mentioned before, z_p and $z_{p'}$ can regarded as the intrinsic and exchangeable value of currency c_x, in Sell-Lead business. Thus, by Eqs. (6), (13) and (15), we have:

$$z_p = z_{p'} \tag{17}$$

or

$$s_{xy} \cdot e_{xo} = ud_{xy} \cdot e_{xo} \tag{18}$$

where all supply and demand of any virtual currency c_x are converted into CONEY. Similarly, for Buy-Lead business and by Eqs. (7), (14) and (16), we have:

$$z_q = z_{q'} \tag{19}$$

or

$$d_{xy} \cdot e_{xo} = us_{xy} \cdot e_{xo} \tag{20}$$

To generalize, we assume that there exists a series of dynamic Pareto exchange points at $t = 1,2,\ldots,k,\ldots,m$. At any time $t = k$, Eqs. (17)–(20) will hold. At $t = 0$, there exists a cutoff point, at which all the demand and supply are initialized to zero such that all elements inside E'_{xo} equals NULL. Thus, at any Pareto exchange points, there will be n exchange rates denoted by the set E'_{xo}. Each element in the set is a variable. Hence, n equations are required to obtain the solution.

Next, we have to set up n equations to find out the value of elements e_{xy} inside E'_{xo}. At all Pareto exchange points $t = k$ for $k \in \{1,\ldots,m\}$, the two equations below can be deduced based on Eqs. (3), (18), and Eqs. (4) and (20).

$$e_{po} \cdot \sum_{x=c_1}^{c_n} s_{px} = \sum_{x=c_1}^{c_n} e_{xo} \cdot s_{xp} \quad (\text{Sell-Lead}) \tag{21}$$

$$\sum_{x=c_1}^{c_n} e_{xo} \cdot d_{xp} = e_{po} \cdot \sum_{x=c_1}^{c_n} d_{px} \quad (\text{Buy-Lead}) \tag{22}$$

In these equations, p denotes an arbitrary currency c_x with $p \in C$. The additional Sell-Lead expression above denotes the total supplied amount from a currency presented in CONEY (at converter's side). This equals the total supplied amounts from other currencies to the currency presented in CONEY (on convertee's side). It can be applied to the Sell-Lead exchange request group, while the additional Buy-Lead expression is for Buy-Lead exchange group. As a result, we are able to build a set of linear equations, as shown in the next two equations, in order to determine the value of the variables $E'_{xo} = \{e_{c_1}, \ldots, e_{c_n o}\}$:

$$\begin{cases} e_{c_1 o} \cdot \sum_{x=c_1}^{c_n} s_{c_1 x} = \sum_{x=c_1}^{c_n} e_{xo} \cdot s_{xc_1} \\ e_{c_2 o} \cdot \sum_{x=c_1}^{c_n} s_{c_2 x} = \sum_{x=c_1}^{c_n} e_{xo} \cdot s_{xc_2} \quad (\text{Sell-Lead}) \\ e_{c_n o} \cdot \sum_{x=c_1}^{c_n} s_{c_n x} = \sum_{x=c_1}^{c_n} e_{xo} \cdot s_{xc_n} \end{cases} \tag{23}$$

$$\begin{cases} \sum_{x=c_1}^{c_n} e_{xo} \cdot d_{xc_1} = e_{c_1 o} \cdot \sum_{x=c_1}^{c_n} d_{c_1 x} \\ \sum_{x=c_1}^{c_n} e_{xo} \cdot d_{xc_2} = e_{c_2 o} \cdot \sum_{x=c_1}^{c_n} d_{c_2 x} \quad (\text{Buy-Lead}) \\ \sum_{x=c_1}^{c_n} e_{xo} \cdot d_{xc_n} = e_{c_1 o} \cdot \sum_{x=c_1}^{c_n} d_{c_1 x} \end{cases} \tag{24}$$

By solving the n equations suggested in the last two equations above, we can obtain the exchange rates $e_{xy} = e_{xo}/e_{yo}$. Let $RET = \{r_1, r_2, \ldots, r_m\}$ be the set of estimated returns gathered from the VMX systems. $r_z \in$ RET is the offer of the corresponding exchange request at each Pareto exchange point $t = k$. The total number of exchange requests is m. By following Eqs. (1) and (2), all of the estimated returns $r_z \in$ RET can be computed and a virtual currency selling and buying offer can be sent back to the players.

6.3. User adjustment on VERA-generated virtual exchange rate

The virtual currency exchange rates generated by matching the supply and demand of virtual currencies through VERA algorithm may fluctuate and this is undesirable for virtual money exchange players. This is because the supply and demand of virtual currencies in each virtual world may also fluctuate due to the different intrinsic world policies and the different utilities of the extrinsic worlds for valuing the currencies. This may prevent individual players from participating in the VMX system since their individual valuation for currency values may be different from the aggregate currency valuation at the Pareto exchange point. To solve this problem, this subsection proposes a user-adjustment mechanism to extend the VERA algorithm with two methods, requests and history.

6.3.1. Request-based method

The first method is to allow players to set a user-adjusted and request-based virtual exchange rate on each exchange request during buying or selling, with the following conditions: (1) The player has the right to select a *minimum acceptable rate MAR* for each exchange request. (2) For any VERA-generated virtual exchange rate *VExR* for the virtual world, if *VExR* < *MAR*, then the *VExR* will be automatically removed from the exchange request list and the unsatisfied request will be pooled into the next round of the VERA computation until *MAR* is reached.

6.3.2. History-based method

The second method is to allow the player to set user-adjusted and history-based virtual exchange rate on all future exchange requests, subject to the following: (1) The player has the right to select a pattern of history rates as a*minimum acceptable rate* (*MAR*) for all exchange requests. (2) For any VERA-generated virtual exchange rates *VExR* for the virtual world, if *VExR* < *MAR*, then *VExR* will be removed automatically from the exchange request list. Then the unsatisfied request will be pooled into the next round of the VERA computation until *MAR* is reached.

These are nearly the same except for the *MAR* that is generated. In the request-based method, *MAR* is directly given by the player. In the history-based method, *MAR*is automatically computed by using the player's preferred pattern. For example, if a player adopts a moving average of historical virtual exchange rates, the VMX system will then present the *MAR* to the player by computing a moving average rate.

The theory behind the two methods is that when system finds that an exchange rate is not desirable at a Pareto exchange point, it will then abandon this Pareto exchange point and force the VMX system to find another Pareto exchange point. The result is that a desirable Pareto exchange point may not be found in the shortest time when trying to satisfy the *MAR* of many players. This will lead to a situation in which the VMX system is in running but no one is willing to sell or buy until new players join in to enable the existence of a Pareto exchange point that is under the current market valuation.

The user-adjustment of *MAR* can effectively satisfy the individual needs of players in virtual money exchanges. It provides a real-time approach to automating the virtual currency exchange calculation

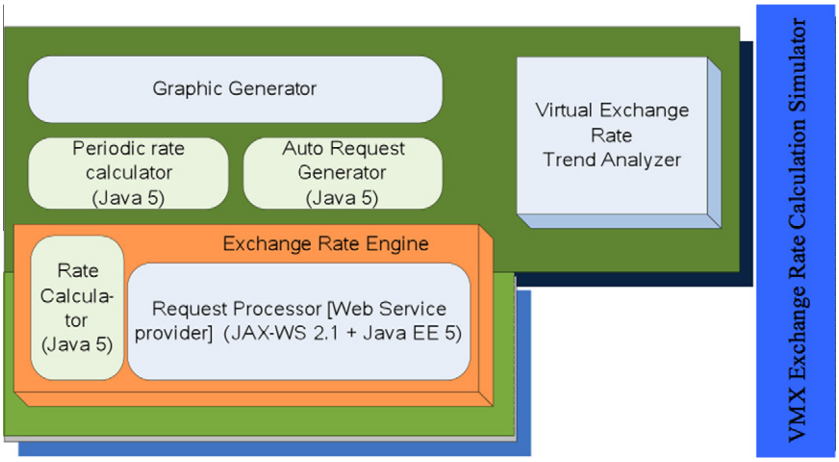


Fig. 5. System components of VMX simulator.

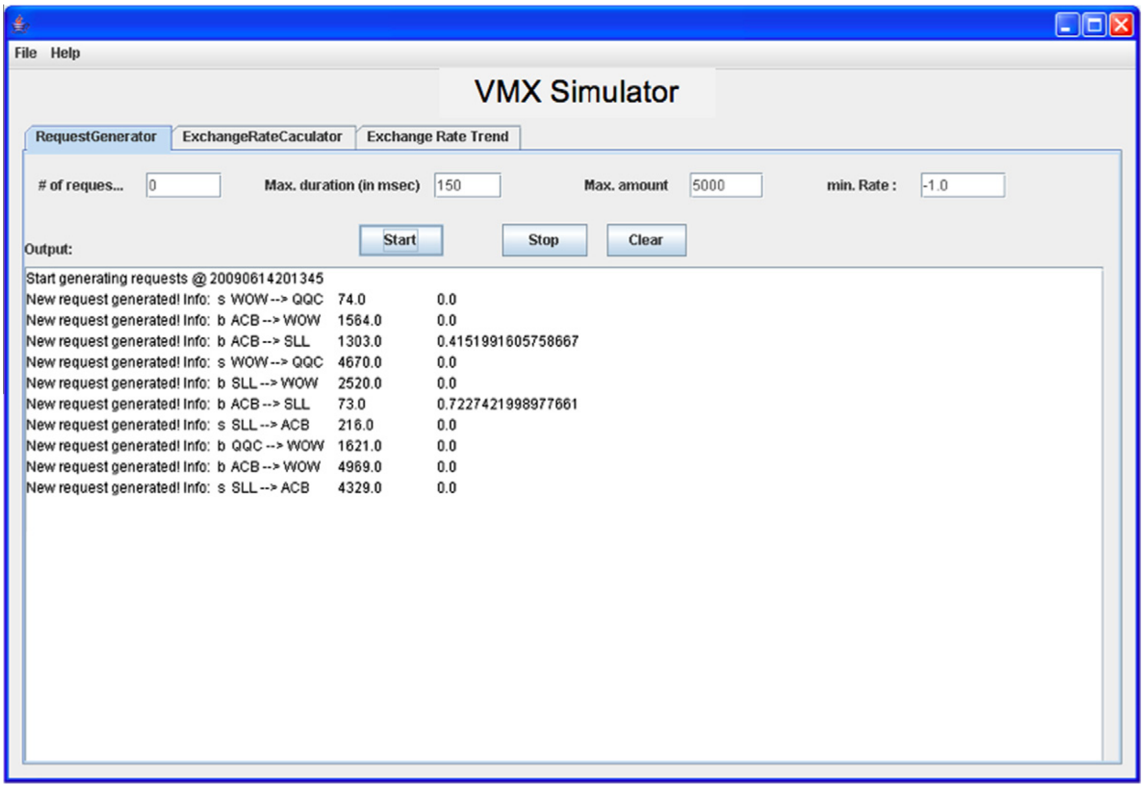


Fig. 6. Auto request generator.

by explicitly telling the players whether their transactions can be fulfilled immediately or if they will be put on a waiting list.

7. VMX Simulator implementation

To provide an experimental mechanism to examine the correctness of the proposed VMX approach, we built VMX simulator. The simulator is based on the VERA algorithm to simulate a variety of virtual currency exchange requests, originating from different players in different virtual worlds. Exchange offers are provided by the VMX system after some point in time. The objective of the tool is to simulate a quantity of requests between randomly triggered virtual currencies at random times, for a random amount of each virtual currency by a random player. Configurable parameters are manually definable in the simulation.

7.1. Components of VMX simulator

The VMX simulator shown in Fig. 5 is implemented to simulate the generation of virtual currency exchange rates. It consists of the components of auto-request generator, a virtual money exchange rate calculator and a trend analyzer. We built an *auto-request generator*, shown in Fig. 6, to randomly generate a certain amount for each exchange request at a random time at a defined acceptable minimum rate. Requests are generated in two ways: the first is to generate a defined number of requests within a defined period of time. The second is to generate requests at a random point in time over some maximum duration of time. Each exchange request can be set with a maximum amount to control the simulation. In addition, two modes of acceptable minimum rate can be set. One is to accept the virtual exchange rate based on each virtual currency's historical moving average rate. The other is to set a player-defined acceptable minimum rate.

An *auto exchange rate calculator*, shown in Fig. 7, calculates exchange rates in a defined timeslot. When the rates are calculated, they are stored in a database. Also, to ensure the calculated exchange rates conform to the VERA strategy that total supply should equal total demand, a simple verification is done to check whether this is true. A summary of transaction results is provided for reference.

A *virtual currency exchange rate trend analyzer*, shown in Fig. 8, provides an intuitive image of the changes in virtual currency exchange rates. The rates are plotted on a graph at each cutoff point. Each currency's exchange rate trend can be seen by tracking its connected lines.

7.2. VMX exchange engine

In the simulator, the VMX exchange engine is most important. It implements the exchange request generator and exchange rate calculator to automatically create simulation results. It implements the VERA algorithm with two functions: process_request and remove_null_supply. The former processes the randomly-generated exchange requests and computes the virtual exchange rates based on the redistribution strategy. The latter considers the players' virtual exchange rate adjustments to remove unsatisfactory requests. To provide a general picture of the VMX exchange engine, the pseudo code for its implementation is shown in Fig. 9.

In this implementation, the virtual currency exchange rate is computed from Eqs. (23) and (24), where all of the e_{xo} are variables that have to be solved in a linear system. One feasible way of solving a linear system is to convert the equations into the form of an augmented matrix and then to apply row reduction. The linear equations are homogeneous, so all of the constant terms are zero. There is always at least one zero (trivial) solution available. However, it is not what we want, as it is meaningless for all of the exchange rates to be zero. To ease computation, the matrix is transformed into *reduced row echelon form* (RREF). By definition though, there are infinitely many solutions if a row of zeros exists,

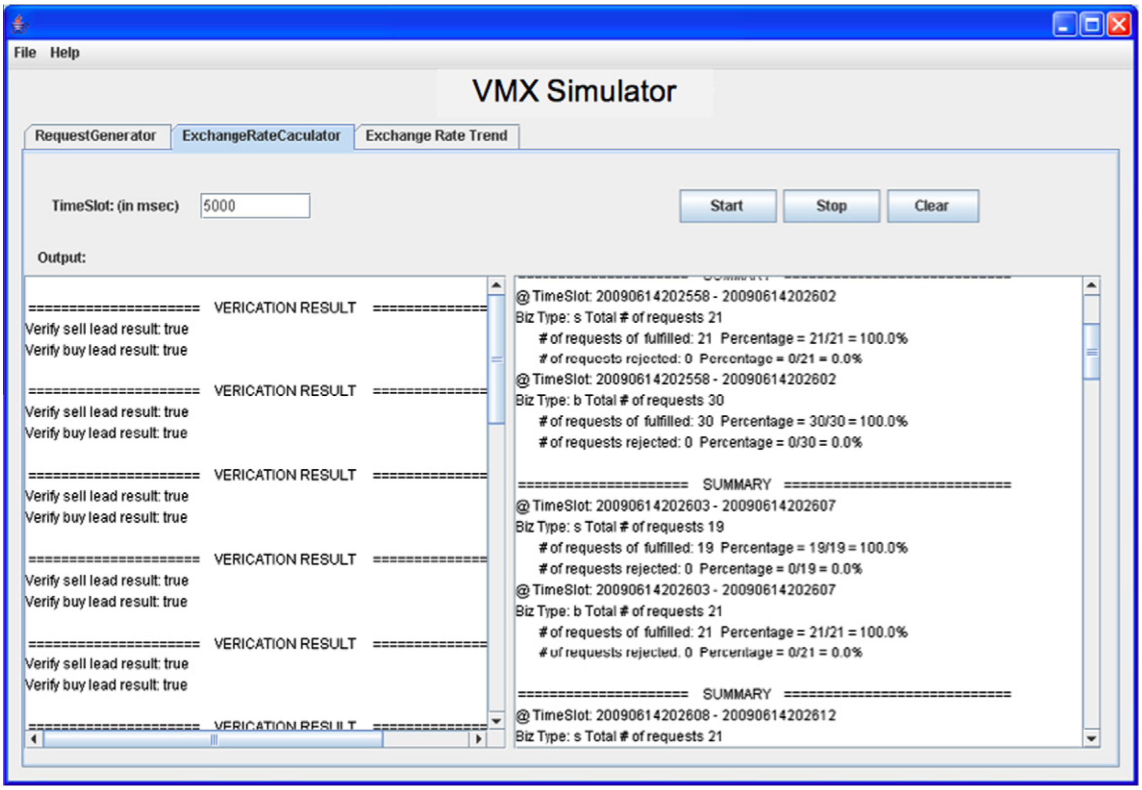


Fig. 7. Auto exchange rate calculator.

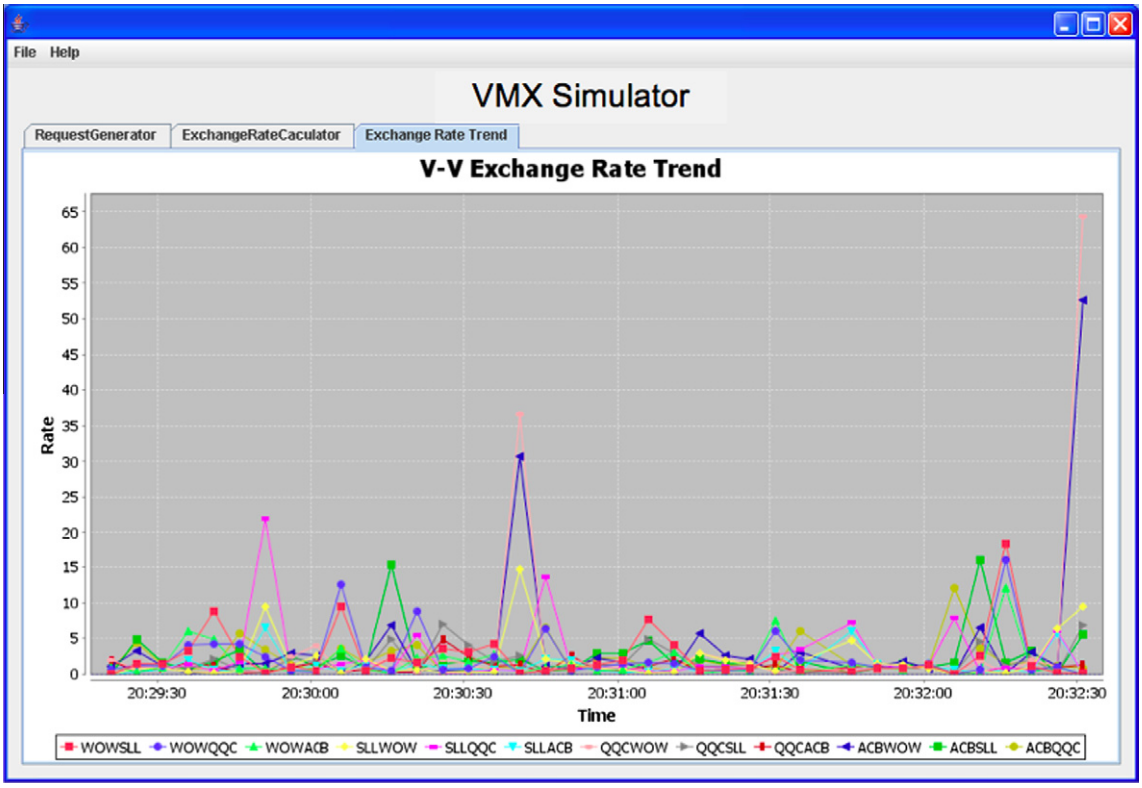


Fig. 8. Trend analyzer.

```
FUNCTION process_request
   BEGIN
      to_be_processed = requests received between starttime and endtime
      DO
               to_be_processed = remove_null_supply(to_be_processed);
                      calculate exchange rates
      WHILE( min. rates of requests in to_be_processed is fulfilled)
              END DO
      Update operation results
END FUNCTION

FUNCTION remove_null_supply(request[] list) RETURN request[]
   VAR  SUPPLY_COUNT[];
   BEGIN
      FOR EACH request in list DO
           Increment SUPPLY_COUNT[index of request.currency_provided]
      END FOR
      FOR EACH element in SUPPLY_COUNT
           IF (SUPPLY_COUNT[element index] == 0)
               to_be_removed = element
               remove requests such that currency_requested =
                                                   to_be_removed from list
           END IF
      END FOR
END FUNCTION
```

Fig. 9. Pseudo code implementing VERA algorithm.

since a free variable exists. This does not imply that there is more than one solution set in VERA though. The earlier results that we obtained are for an exchange rate between an arbitrary virtual currency and an intermediate virtual currency, CONEY. The actual rate we are looking for relative exchange rate between two virtual currencies, c_x and c_y, which can be obtained via $e_{xy} = e_{xo}/e_{yo}$. Thus, the value of e_{xo} never has to be computed.

Fig. 10 shows an example of a matrix in RREF form, and b_n with $n \in \{1,2,3,4,5\}$ indicates a constant value.

Assume the variables of the linear system $\{e_{1o}, e_{2o}, e_{3o}, e_{4o}, e_{5o}\}$. We can obtain four relative exchange rates from the above matrix, which are given by $e_{14} = e_{1o}/e_{4o} = -b_1$, $e_{24} = e_{2o}/e_{4o} = -b_2$, etc. Other unknown variables can be obtained by the inversion and transformation properties of the exchange rate, as stated below in two properties:

- *Exchange Rate Property 1 (Inversion):* $e_{xy} = 1/e_{yx}$
- *Exchange Rate Property 2 (Transformation):* $e_{xy} = e_{xk}/e_{yk}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & b_1 & 0 \\ 0 & 1 & 0 & 0 & b_2 & 0 \\ 0 & 0 & 1 & 0 & b_2 & 0 \\ 0 & 0 & 0 & 1 & b_4 & 0 \\ 0 & 0 & 0 & 0 & b_5 & 0 \end{bmatrix}$$

Fig. 10. A matrix in RREF.

After applying Property 1 and 2 in our calculation, all of the exchange rates can be obtained. Fig. 11 shows the steps for making these calculations of the exchange rates. Once the exchange rates have been computed, the exchange offer can be calculated and the corresponding transaction entry can be updated.

The VMX simulator implemented in this section is useful to simulate the process of virtual currency exchange rate generation, and offers a useful way of evaluating the strategies used in the VMX approach.

8. Experiments with the VMX simulator

We next evaluate the VMX approach by conducting simulation experiments with its algorithm. In the experiments, we will set up a series of test cases with specific durations of time to examine how the VERA algorithm performs under a number of different scenarios.

8.1. Experimental setting and assumptions

Besides the examination of the correctness of the VERA algorithm, one of the most important objectives of the experiments is to check how the players' minimum acceptable exchange rates influence the usable virtual currency exchange rates that are adopted for virtual currency exchange.

To accommodate the simulation for experiments, the virtual currency exchange requests are randomly and independently generated with a series of parameters that are set by the VMX simulator players (actually a module representing their actions). It is also assumed that there is no prior knowledge of the exchange request patterns within the system. The amount of supply and demand related to the currency involved in each exchange request is random and bounded by some predefined upper limit.

8.2. Experimental scenarios

The experiment consists of sixteen scenarios. Each scenario takes approximately five minutes of running time within the simulator. Table 1 shows the value of each parameter set for the scenarios. These parameters are: total number of virtual currencies (*Virtual Currency Number*), maximum amount per exchange request (*Maximum Amount*), minimum accepted rate set by exchange Player (*Minimum Accept Rate*), maximum time duration in milliseconds representing the timeslot between two exchange requests that are generated (*Timeslot between Requests*), and a timeslot in milliseconds to calculate a new exchange rate after the last calculation has been completed (*Timeslot for Calculation*).

8.3. Experimental results

Table 2 shows the results of the experiment based on the parameters described in Table 2. Its key inputs include the total number of requests (*Total Requests*), the average number of requests generated per timeslot (*Average Number of Requests*), the total number of timeslots per experiment (*Number of Timeslots*),

1. Prepare virtual currency request data;
2. Convert into a form conforming to VERA;
3. Simplify the matrix to RREF matrix;
4. Calculate the relative virtual currency exchange rates;
5. Store result.

Fig. 11. Steps for calculating virtual currency exchange rates.

Table 1
Parameter values of each scenario.

Experiment scenario number	Virtual currency number	Maximum amount per exchange request	Minimum accepted rate set by exchange player	Maximum duration of timeslot between two exchange requests (ms)	Timeslot to calculate a new exchange rate after last calculation (ms)	Number of timeslots in each experiment	Duration of experiment (s)
1	4	100	0	100	5000	65	325
2	4	100	0	1000	5000	65	325
3	4	100	0	1000	10,000	32	320
4	4	10,000	0	100	5000	65	325
5	4	1,000,000	0	100	5000	64	320
6	4	100	Avg	100	5000	71	355
7	4	100	Avg	100	10,000	33	330
8	4	100	Avg	100	30,000	14	520
9	4	100	0.5	100	5000	66	330
10	4	100	0.1	100	5000	74	370
11	8	1000	0	100	5000	64	320
12	8	1000	0	100	10,000	32	320
13	4	10	Avg	10	5000	79	395
14	4	100	Avg	100	5000	36	180
15	4	100	Avg	100	5000	62	310
16	4	100	Avg	100	5000	28	140

Table 2
Experiment results.

Experiment number	Number of timeslots per experiment	Duration of experiment (s)	Total number of requests	Average number of requests per timeslot	Results		
					Accept rate (%)	Reject rate (%)	Range of rate
1	65	325	2623	40.35	100	0	[0,40.07]
2	65	325	315	4.85	42	58	[0,20.5]
3	32	320	300	9.38	81	19	[0,1219.56]
4	65	325	2610	40.15	100	0	[0,27.02]
5	64	320	1664	26.00	99	1	[0,56.17]
6	71	355	2765	38.94	6	94	[0,2.82]
7	33	330	2439	73.91	4	96	[0,3.77]
8	14	520	2575	183.93	7	93	[0,3.6]
9	66	330	1614	24.45	32	72	[0,3.13]
10	74	370	1858	25.11	97	3	[0,18.39]
11	64	320	1494	23.34	80	20	[0,253.65]
12	32	320	1466	45.81	96	4	[0,63.18]
13	79	395	4831	61.15	7	93	[0,3]
14	36	180	930	25.83	0	100	[0,1.4]
15	62	310	1650	26.61	58	42	[0,10]
16	28	140	752	26.86	0	100	[0,0]

Table 3
Relationship between minimum accept rate and percentage of accept rates.

Experiment number	1	2	3	4	5	11	12	10	9	6	7	8	13	14	15	16
Minimum rate	0	0	0	0	0	0	0	0.1	0.5	Avg.	Avg.	Avg.	Avg.	Avg.	Avg.	Avg.
Accept rate (%)	100	42	81	100	99	80	96	97	32	6	4	7	7	0	58	0

and the total number of seconds for the experiment's duration (*Number of Seconds*). The output (*Results*) includes the accept and reject rates of the exchange requests within a timeslot in which the exchange rate calculation is made.

The experiment results show that Cases 6, 7, 8, 13, 14 and 16 are not satisfactory. The accept rates of these scenarios are less than 10%. The results are better for Cases 1, 4, 5, 10 and 12 with an accept rate of over 95%. Cases 2, 3, 9, 11 and 15 have accept rates that are between 10% and 95%. Among these experimental cases, the accept rate has an inverse proportionate relationship with the minimum accepted rates that players have set. When the minimum acceptable rate is set from zero to the moving average of the historical rates, the accept rate changes; it goes from a higher accept rate to a lower accept rate. See Table 3.

Another observation is that an increase in the maximum allowed amount of each exchange request does not affect the results too much, as shown in Experiments 1, 4, 5, 11 and 12. Additionally, an increase in the number of virtual currencies affects the accept rate that is generated, provided that all of the other variables are unchanged. In additional experiments that we conducted that are not included in the above scenarios, we observed a 20% drop in the accept rate when the number of virtual currencies to be exchanged is doubled. The losses can be recovered when the timeslot of the exchange rate calculation is made longer.

8.4. Explanation of the experimental results

The inverse proportionate relationship between the minimum acceptable rate set by the simulator's components for the players and the accept rate for the exchange transactions that the simulator makes indicates that the generated virtual exchange rates directly relate to the players' requirements. The simulator requires more time to effectively match their required virtual currency exchange rates. This affects the players' waiting times for exchange transactions to be completed within the simulation.

Second, when the simulator uses the historical moving average exchange rate of a virtual currency as the minimum rate, the percentage of accept rates is drastically lower. This can be explained by using the trend analyzer. It reveals that the simulator cannot find a predictable pattern for each virtual currency exchange rate. See Fig. 8. A possible explanation is that, in the simulator, there is no expectation that players achieve any particular level of wealth in the different virtual worlds. Thus the exchange requests are randomly generated without any patterns, as opposed to being generated on the basis of some observed patterns that have their own underlying logic related to the inhabitants' desire to maximize their wealth. This again leads us to conclude that the history-based minimum accept rates may be too strict when the historical exchange rates does not demonstrate a logical pattern. The implication here is that this rate might be more appropriately adopted only if the players' transaction-making patterns can be predicted on the basis of observed supply and demand, or some other observable and meaningful indicators.

Another possibility is that when players make some evaluations about their business performance in different virtual worlds, their virtual currency exchange requests will show certain types of discernible patterns. This is likely to increase the percentage of accept rates and make the history-based minimum exchange rate more useful in the simulation.

9. Discussion

The lack of virtual money exchange technology to protect virtual wealth in the operation of virtual worlds on the Internet motivated us to develop a virtual money exchange system and to design a novel virtual money exchange rate algorithm. After scrutinizing various approaches and considering the critical comments of colleagues and reviewers, we identified several theories that become candidates for providing a basis for the design of the VERA algorithm. The first is Pareto optimization theory (Pareto 1906), which we adopted in our research. The second is the Arrow-Debreu (1954) model for the existence of a general equilibrium. The third is double auction theory (Smith 1962, Gode and Sunder 1993, Cliff and Bruten 1997).

Double auction theory was established on the basis of non-Walrasian equilibrium, in which a series of designated prices are used as inputs to determine decisions on commodity trades according to individual preferences or willingness. This is similar to a game in which someone may lose out when Pareto optimality has not been achieved. Another observation is related to the simulation approach that we have used. In our simulation, the virtual currency exchange rates, which are the prices established between any two currencies in the sense of a double auction, are the outputs of that are produced from the virtual currency exchange function. In our simulator, the VERA algorithm tries to produce an exchange rate price as an output based on the related input quantities of the currencies. This causal relationship between price and quantity suggests that the VERA algorithm cannot be implemented on the basis of double auction mechanism design theory.

The Arrow-Debreu model relies on Pareto optimality theory to prove the existence of a general equilibrium. It is similar to what we have adopted, but not identical. Why? Because for a feasible allocation $\sum_{i=1}^{M} x_i = \sum_{i=1}^{M} \omega_i + \sum_{j=1}^{J} y_j$ of the Arrow-Debreu model. In this expression, x_i is the consumption of ith consumer of consumption set M, ω_i is the ith endowment of consumption set M, and y_j is the production by the jth producer of production set J. We further note in our virtual currency exchange context that no production occurs, that the input X would be the quantity of virtual currency demanded which cannot be negative, and, that the endowment ω would be the supply quantity of virtual currency.

Thus, our VMX modeling approach seems like it might be similar to the simple trading economy represented by the Arrow-Debreu model. Further investigation suggested to us that there are some problems with this analogy though. For example, we might ask: What determines the quantity of virtual currency demand? What does an endowment consist of in this context? And how will it be determined? Our evaluation suggests a number of conclusions. First, the word "endowment" is not an appropriate term to describe the supply of virtual currency quantity, because there are no gifts or free resources in virtual worlds that use virtual currencies. Second, the virtual currency supply quantity will be determined by the labor time of virtual inhabitants in the different virtual worlds. This quantity could be arbitrarily large or small, but its intrinsic value will be fixed at some point in time. (3) The virtual currency demand quantity of extrinsic worlds can be also represented by a larger or smaller number as the supply quantity, but its exchangeable value will be determined based on trust and utility relative to the virtual currencies that are being exchanged. This value will also need to be fixed at some point in time.

From the discussion above, we can see that the proposed VMX model seeks to establish equivalence between the intrinsic value of the virtual currencies in intrinsic worlds and the exchangeable value of the currencies across extrinsic worlds. This is a basis for achieving fair virtual currency exchange. Note that we have not concerned ourselves with the quantities of intrinsic value and exchangeable value. The introduction of Pareto optimal exchange should enable the determination of what amount of intrinsic value should be equal in exchangeable value terms to derive a fair virtual money exchange regime. This is what the VERA algorithm that we have proposed has achieved, and we be believe that it is a novel contribution.

10. Conclusion

In this article, we proposed a new approach to virtual money exchange to protect virtual wealth by satisfying a continuous set of Pareto optimal exchange points. With this approach, we were able to address the importance of protecting virtual wealth, after reviewing the existing research on virtual money, and discussing the difficulty of evaluating the total value of such currencies in the virtual world. To prove that the existence of a virtual money exchange market is possible, we first proved the basis for the existence of a common value system wherein the total intrinsic value of a virtual currency is equal to its total exchangeable value at any Pareto optimal exchange point. To protect virtual wealth yet maintain fair exchange of virtual currencies between virtual worlds, we proposed the VMX approach. This approach leverages a redistribution strategy so that the intrinsic value of the money supply must be equal to the exchangeable value of the money demand at any Pareto optimal exchange point. This strategy enables virtual currencies to be exchanged in a fair way, and allowed us to create a technical design for the proposed VMX system using a novel VERA algorithm. The proposed system dynamically generates virtual currency exchange rates at a series of Pareto optimal exchange points that match intrinsic values and exchangeable values for different virtual currencies.

The proposal of the VERA algorithm is an important result of this research, so we have included a proof that it is correct in Appendix A. To address the high fluctuations of virtual currency exchange rates that participants in virtual worlds might not favor, we extended the VERA algorithm to include two types of user adjustments to the minimum acceptable exchange rates. To provide a platform to observe virtual money exchange behaviors, we also designed and implemented a VMX simulator. We conducted sixteen experiments to derive results utilizing this simulator. The results show that setting a minimum acceptable exchange rate has a big impact on the success and failure of the virtual money exchange process. To establish the theoretical foundation for our VMX approach, we drew upon three theories for optimization: the Arrow-Debreu model, double auction equilibrium theory, and Pareto optimality theory. The last of these three proved to be the most effective for our purposes.

This article represents a pioneering research effort in the area of protecting virtual wealth through virtual money exchange. It makes contributions to the areas of e-commerce and virtual worlds by providing: a better understanding of virtual wealth relevant to its protection; new knowledge on virtual money and its creation, forms and uses; and proof of the existence of a common value system in which the total intrinsic value equals the total exchangeable value of a virtual currency, which supports the operation of a virtual money exchange. This research also offers: a redistribution strategy for generating virtual currency exchange rates based on the Pareto optimal exchange points of intrinsic value and exchangeable value of virtual currencies; the VERA algorithm for dynamically generating virtual currency exchange rates; and finally, the design and implementation of a technology artifact that supports the simulation of various user behaviors for generating virtual currency exchange rates. More research involving the simulation of virtual world currency exchange rate construction needs to be conducted, as well as to reconcile the contradiction between generating fair virtual currency exchange rates while providing the basis for less fluctuation of virtual currency exchange rates.

Acknowledgments

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Appendix A. Correctness proof of the VERA algorithm

In this Appendix, we will prove that the computational method of the VMX Exchange Rate Algorithm (VERA) is correct. As we stated in the article, the strategy of the virtual exchange rate regime is redistribution. That is:

- *Given that the exchangeable value and intrinsic value of an arbitrary virtual currency are equal, then the total demand of an arbitrary virtual currency, in terms of its money value, and in the units of that virtual currency, can be fulfilled by the total supply of that virtual currency, in terms of money value, and in the units of that virtual currency.*
  The above statement formulates the *proposition*of the proof. It can be interpreted as a formal Proposition as follows:
- Proposition 2:

$$(EV_{c_k} = IV_{c_k}) \Rightarrow \left(\sum_{y=c_1}^{c_n} ud_{c_k y} = \sum_{y=c_1}^{c_n} s_{c_k y}\right)$$

EVc_k denotes the exchangeable value of a virtual currency c_k with $c_k \in C$. Likewise, IVc_k denotes the intrinsic value of the currency c_k. $ud_{c_k y}$ represents unknown demand for c_k in terms of money value, while $s_{c_k y}$ is the supply of c_k in terms of money value. The notation $p \Rightarrow q$ denotes a *conditional implication relationship*indicating that the truth of *p*implies the truth of *q*.

We will apply mathematical induction to prove the above proposition. A Pareto optimal exchange point must be reached for this to be proved. Since the VMX system consists of different states, the proposition should be valid for all states. Therefore, the proof is separated into two parts: (A1) the proof of the proposition by mathematical induction; and (A2) the proof of Pareto optimal exchange point at $t = 0$, $t = 2$, and $t = m$.

A.1. Proof by mathematical induction

First, we develop a proof of the proposition by induction. The complete proof should consist of both the Sell-Lead and Buy-Lead. Due to page limitations, we will only show the proof of Sell-Lead. The proof of Buy-Lead can be constructed in a similar fashion.

Let $f(n) := (EV_{c_k} = IV_{c_k}) \Rightarrow \left(\sum_{y=c_1}^{c_n} s_{ky} = \sum_{y=c_1}^{c_n} ud_{ky}\right)$, where the pre-condition follows Theorem 2.

Part A. When $n = 1$, the following will always be true:

1. $\because ud_{c_1 c_1} = s_{c_1 c_1} \cdot e_{c_1 c_1}$ (Eq. (3))
2. $\because ud_{c_1 c_1} = s_{c_1 c_1}$ $(e_{c_1 c_1} = 1)$
3. $\because EV_{c_1} = IV_{c_1}$ (Theorem 2)
4. $\because (EV_{c_1} = IV_{c_1}) \Rightarrow \sum_{y=c_1}^{c_1} ud_{c_1 y} = \sum_{y=c_1}^{c_1} s_{c_1 y}$

Part B. Assume that when $n = m$, the following is true:

$$(EV_{c_k} = IV_{c_k}) \Rightarrow \left(\sum_{y=c_1}^{c_m} ud_{c_k y} = \sum_{y=c_1}^{c_m} s_{c_k y}\right). \quad \text{(Proposition 2)}$$

Part C. When $n = m + 1$,

1. $\because EV_{c_k} = IV_{c_k}$
2. $\therefore s_{c_{m+1} c_k} \cdot e_{(m+1)o} = s_{c_k c_{m+1}} \cdot e_{ko}$ (Eq. (21))
3. $\because s_{c_{m+1} c_k} = ud_{c_k c_{m+1}} / e_{(m+1)k}$ (Eq. (3))
4. $\therefore s_{c_{m+1} c_k} \cdot e_{(m+1)o} = (ud_{c_k c_{m+1}} / e_{(m+1)k}) \cdot e_{(m+1)o}$
5. $\therefore s_{c_{m+1} c_k} \cdot e_{(m+1)o} = ud_{c_k c_{m+1}} \cdot e_{ko}$ (Eq. (12))
6. $\therefore s_{c_k c_{m+1}} \cdot e_{ko} = ud_{c_k c_{m+1}} \cdot e_{ko}$ (Steps 2 and 5, by substitution)
7. $\therefore s_{c_k c_{m+1}} = ud_{c_k c_{m+1}}$
8. $\because \sum_{y=c_1}^{c_m} ud_{c_k y} = \sum_{y=c_1}^{c_m} s_{c_k y}$ (Proposition 2)
9. $\therefore \sum_{y=c_1}^{c_m} ud_{c_k y} + ud_{c_k c_{m+1}} = \sum_{y=c_1}^{c_m} s_{c_k y} + s_{c_k c_{m+1}}$ (Steps 7 and 8, equality, addition)
10. $\therefore \sum_{y=c_1}^{c_{m+1}} ud_{c_k y} = \sum_{y=c_1}^{c_{m+1}} s_{c_k y}$
11. $\therefore (EV_{c_k} = IV_{c_k}) \Rightarrow \sum_{y=c_1}^{c_{m+1}} ud_{c_k y} = \sum_{y=c_1}^{c_{m+1}} s_{c_k y}$

Thus, Proposition 2 holds when $n = m + 1$.

There are several aspects that are worth emphasizing: (1) The *intrinsic value of currency* c_k is defined to be the supply of c_k, that is, the amount of that is "leaving" the virtual world of c_k. (2) The *exchangeable value of currency* is defined to be equal to the supply of the other currency that is to be exchanged for c_k, that is, the amount of non-currency that is "entering" the virtual world of c_k. The total amount also equals the unknown demand for c_k. (3) The introduction of CONEY as a common value system serves to ensure that all virtual currencies are denominated in the same units. As a result, the equality between the intrinsic value and the exchangeable value of currency c_k implies the corresponding values in the units of CONEY will be equal. □

A.2. Proof of Pareto exchange point at t = 0, t = 1, and t = m

Since Proposition 2 have been proven to be valid by induction, the correctness of the VERA algorithm has to be proved to hold for all $t = k$, where k is a non-negative integer. Alternatively, we can prove this by contradiction. Assume Proposition 2 is not true for $t = r$ such that *r*is a non-negative integer. Thus, this implies that $\sum_{y=c_1}^{c_n} s_{ky} \neq \sum_{y=c_1}^{c_n} ud_{ky}$ and $EV_{ck} \neq IV_{ck}$ as a consequence. This is a contradiction with Theorem 2 that $EV_{ck} = IV_{ck}$. Therefore, Proposition 2 is proved to be valid. □

Appendix B. A notation table

$\mathcal{A}$	A finite set of computer agents that act on behalf players who request to exchange virtual currencies between different virtual worlds
$\mathcal{V}$	A value set
$\leqslant$	A partial order
$\mathcal{S}$	A Cartesian product for an exchange space with $\mathcal{S} = \mathcal{V}_1 \times \mathcal{V}_2 \times \cdots \times \mathcal{V}_k \times \cdots \times \mathcal{V}_n$
$\equiv$	Pareto equivalent
E^*	An exchange with $e^* = (e_1, e_1, \ldots, e_k, \ldots, e_n)$
E	An exchange set containing e^* elements
C	A finite set of virtual currencies with $C = \{c_1, c_2, \ldots, c_n\}$
c_x, c_y	Any virtual currencies in the set of virtual currencies
c_o	The intermediate common value virtual currency, CONEY
d_{xy}	The demand for c_x; associated with obtaining virtual currency c_x using virtual currency c_y
ud_{xy}	The unknown demand for current c_x; the quantity is unknown before a quantity of currency c_y is set
s_{xy}	The supply of c_x; enables the supply of virtual currency c_x for virtual currency c_y
us_{xy}	The unknown supply for currency c_x; the quantity is unknown before a quantity of currency c_y is set
e_{xy}	A virtual currency exchange rate for virtual currency c_x to virtual currency c_y
E_{xy}	A set of virtual currency exchange rates with $E_{xy} = \{e_{11}, e_{12}, \ldots, e_{1n}, \ldots, e_{nn}\}$
e_{xo}	A virtual currency exchange rate of virtual currency c_x to CONEY
$\mid X \mid$	The cardinality of set X
IV	The intrinsic values of a set of virtual currencies C
EV	The exchangeable values of a set of virtual currencies C
IVc_x	The intrinsic value of a virtual currency c_x
EVc_x	The exchangeable value of a virtual currency c_x
NULL	Non-existent

(*continued on next page*)

t	Time
z_p	The CONEY supply converted from the supply for virtual currency c_x
z_q	The CONEY demand converted from the demand for virtual currency c_x
MAR	The minimal acceptable exchange rate of a player for any virtual currency exchange rate

References

Alther, R., Cherakov, L., and Becker, C. Virtual spaces: enabling immersive collaborative enterprise: part 2: implementation and lessons learned. IBM Developer Works, Armonk, NY, 2009. <www.ibm.com/developerworks/webservices/library/ws-virtualspaces2/index.html>.

Arrow, K., and Debreu, G. Existence of an equilibrium for a competitive economy. *Econometrica*, 22, 3, 1954, 265–290.

Bentham, J. An Introduction to the Principles of Morals and Legislation, digital edition, 1781. <www.constitution.org/jb/pml.htm>.

Budd, L. Offdigital: why money has always been virtual. *City*, 4, 1, 2000, 155–161.

Cherbakov, L., Brunner, R., Smart, R., and Lu, C. Virtual spaces: enabling immersive collaborative enterprise, part 1: introduction to the opportunities and technologies. IBM Developer Works, Armonk, NY, 2009. <www.ibm.com/developerworks/webservices/library/ws-virtualspaces>.

Cikic, S., Grottke, S., Lehmann-Grube, F., and Sablatnig, J. Cheat-prevention and analysis in online virtual worlds. In Proceedings of the 1st ICST International Conference on e-Forensic Applications and Techniques in Telecommunications, Information and Multimedia, Adelaide, Australia, January 21–23, 2008.

Cliff, D., and Bruten, J. Less than human: simple adaptive trading agents for CDA markets. HP Labs technical report HPL-97-155, Hewlett-Packard Research Laboratories, Bristol, UK, 1997.

Geilen, M., Basten, T., Theelen, B., and Otten, R. An algebra of Pareto points. In *Proceedings of the 5th International Conference on Application of Concurrency to System Design, St. Malo, France, June 7–9, 2005*, IEEE Computer Society Press, Los Alamitos, CA, 2005.

Gode, D., and Sunder, S. Allocative efficiency of markets with zero-intelligence traders: market as a partial substitute for individual rationality. *Journal of Political Economy*, 101, 1, 1993, 119–137.

Guo, J. Inter-enterprise business document exchange. In *Proc. International Conference on Electronic Commerce, Fredericton, New Brunswick, Canada, August 14–16, 2006*, ACM Press, New York, NY, 2006, 427–437.

Guo, J. *Collaborative Concept Exchange*. VDM Publishing, Saarbrücken, Germany, 2008.

Guo, Y., and Barnes, S. Why people buy virtual items in virtual worlds with real money. *The DATA BASE for Advances in Information Systems*, 38, 4, 2007, 69–76.

Guo, J., and Chow, A. Virtual money systems: a phenomenal analysis. In *Proceedings of the 2008 IEEE Joint Conference on E-Commerce Technology and Enterprised Computing, E-Commerce and E-Services,Washington, DC, July 22–24, 2008*, IEEE Computer Society Press, Los Alamitos, CA, 2008, 267–272.

Guo, J., Chow, A., Gong, Z., and Sun, C. Virtual wealth realization in virtual and real worlds. In *Proceedings of the 9th International Conference on E-Business Engineering, Macau, China, October 21–23, 2009*, IEEE Computer Society Press, Los Alamitos, CA, 2009, 85–94.

Hauben, M., and Hauben, R. Netizens: an anthology. Columbia University, New York, NY, June 12, 1996. <www.columbia.edu/~rh120/>.

Ives, B., and Junglas, I. APC Forum: business implications of virtual worlds and serious gaming. *MIS Quarterly Executive*, 7, 3, 2008, 151–156.

Kumar, S., Chhugani, J., Kim, C., Kim, D., Nguyen, A., Dubey, P., Bienia, C., and Kim, Y. Second Life and the new generation of virtual worlds. *Computer*, 41, 9, 2008, 46–53.

Lee, F., Vogel, D., and Limayem, M. Virtual community informatics: what we know and what we need to know. In R. Sprague (ed.), *Proceedings of the 35th Hawaii International Conference on Systems Science, Waikoiloa, HI, January 2002*, IEEE Computer Society Press, Los Alamitos, CA, 2002, 2863–2872.

Mackenzie, R. Virtual money, vanishing law: dematerialisation in electronic funds transfer, financial wrongs and doctrinal makeshifts in English legal structures. *Journal of Money Laundering Control*, 2, 1, 2007, 22–32.

Marx, K. Capital (Volume 1), digital edition, 1887. <www.marxists.org/archive/marx/works/cw/volume35/index.htm>.

Mill, J. The Collected Works of John Stuart Mill (Volume II), The Principles of Political Economy with Some of their Applications to Social Philosophy (Books I–II), digital edition. Online Library of Liberty, 1848a. <oll.libertyfund.org/title/102>.

Mill, J. The Collected Works of John Stuart Mill (Volume III), Principles of Political Economy Part II, digital edition. Online Library of Liberty, 1848b. <oll.libertyfund.org/title/243.

Mill, J. Essays on Ethics, Religion, and Society, digital edition. Online Library of Liberty, 1883. <oll.libertyfund.org/title/241>.

Pareto, V. Manual of Political Economy. Macmillan, London, UK, 1906 (Translated by Ann S. Schwier and Alfred N. Page, Augustus M. Kelley, New York, NY, 1971).

Peng, H., and Sun, Y. Network virtual money evolution mode: moneyness, dynamics and trend. In *Proceedings of the 2009 IEEE International Conference on Information and Automation, Zhuhai, Macau, China, June 22–25, 2009*, IEEE Computing Society Press, Los Alamitos, CA, 2009, 550–555.

Peng, H., and Xu, X. The analysis of virtual money demand under one-way exchange mechanism. In *1st International Workshop on Education Technology and Computer Science, Wuhan, Hubei, China, 2009*, IEEE Computing Society Press, Los Alamitos, CA, 2009, 719–723.

Quiané-Ruiz, J., Lamarre, P., Cazalens, S., and Valduriez, P. Managing virtual money for satisfaction and scale up in P2P systems. In *Proceedings of the 2008 International Workshop on Data Management in Peer-to-Peer Systems, Nantes, France, March 2008*, ACM Press, New York, NY, 2008, 67–74.

Rheingold, H. *The Virtual Community: Homesteading on the Electronic Frontier*. Harper Perennial, New York, NY, 1993.

Ricardo, D. *The Principles of Political Economy and Taxation*. Dent and Dutton, London, UK, 1912.

Smith, A. Wealth of Nations, MεταLibri, digital edition, 1776. <metalibri.wikidot.com/authors:adam-smith>.

Smith, V. L. An experimental study of competitive market behaviour. *Journal of Political Economy*, 70, 2, 1962, 111–137.

Solomon, E. *Virtual Money: Understanding the Power and Risks of Money's High-Speed Journey into Electronic Space*. Oxford University Press, New York, NY, 1997.

Wang, Y., and Mainwaring, S. Human–currency interaction: learning from virtual currency use in China. In *Proceedings of the 26th International Conference on Human Factors in Computing Systems, Florence, Italy, April 5–10*, ACM Press, New York, NY, 2008, 25–28.

Wigand, R. T. Electronic commerce: definition, theory and context. *The Information Society*, 13, 3, 1997, 1–16.

Yamaguchi, H. An analysis of virtual currencies in online games. Working paper, Japan Center for International Finance, International Christian University, September 1, 2004. <http://www.ssrn.com/abstract=544422>.